Fuzzy-Cuts: A Knowledge-Driven Graph Based Method for Medical Image Segmentation

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Abstract

Image segmentation is, in general, an ill-posed problem and additional constraints need to be imposed in order to achieve the desired results. Particularly, in the field of medical image segmentation, a significant amount of prior knowledge is available that can be used to constrain the solution space of the segmentation problem. However, most of this prior knowledge is, in general, vague or imprecise in nature, which makes it very difficult to model.

In this poster, we present Fuzzy-Cuts, a knowledge-driven, graph-based method for medical image segmentation. We cast the problem of image segmentation as the Maximum A Posteriori (MAP) estimation of a Markov Random Field (MRF) which, in essence, is equivalent to the minimization of the corresponding Gibbs energy function. Considering the inherent imprecision that is common in the a priori description of objects in medical images, we propose a fuzzy theoretic model to incorporate knowledge-driven constraints into the MAP-MRF formulation. In particular, we focus on prior information about the object's location, appearance and spatial connectivity to a known seed region inside the object. To that end, we introduce fuzzy connectivity and fuzzy location priors that are used in combination to define the 1st-order clique potential of the Gibbs energy function.

In our experiments, we demonstrate the application of Fuzzy-Cuts to the problem of heart segmentation in non-contrast computed tomography (CT) data.

Formulation of Segmentation Problem

Image Segmentation Problem:

> *"find a mapping $f: P \rightarrow L$ that minimizes some energy functional $E(f \mid D)$ conditioned over the observed image data D, where P is the set of pixels and L is the set of labels."*

We cast the segmentation problem as a MAP-MRF problem. We compute the *MAP-MRF solution* by minimizing the following ***Gibbs energy function***:

$$E(f \mid D) = \sum_{i \in P} V_i(f_i \mid D) + \sum_{i \in P} \sum_{j \in N_i} V_{ij}(f_i, f_j \mid D)$$

1st-order Clique Potential - $V_i(f_i \mid D)$

Measures the cost of assigning the label f_i to the pixel i given prior knowledge about the data D.

We define $V_i(f_i \mid D)$ using a fuzzy theoretic model that unifies various types of prior knowledge.

We consider prior information about the object's location, appearance and spatial connectivity to a known seed region inside the object.

Formally, we define $V_i(f_i \mid D)$ as a spatial fuzzy set on the image space S as shown below:

$$V_i(f_i \mid D) = c\left(t\left(\mu_{fcon}^{O_{f_i}}(i), \mu_{floc}^{O_{f_i}}(i)\right)\right)$$

Fuzzy Connectivity Prior - $\mu_{fcon}^{O_{f_i}}(i)$

Notion of ***fuzzy connectedness***:

> *"if two regions have similar appearance and if they are spatially connected in the image space then they most likely belong to the same object"*

Given a seed region R, we define $\mu_{fcon}^{O_{f_i}}(i)$ as a spatial fuzzy set representing a fuzzy connected component of the object O_{f_i} as shown below:

$$\mu_{fcon}^{O_{f_i}}(i) = \max_{p_{Ri} \in P_{Ri}} \left\{ \min_{1 \le j < |p_{Ri}|} \left[\psi_{aff}^{O_{f_i}}(j, j+1) \right] \right\}$$

$\psi_{aff}^{O_{f_i}}(p,q)$ is the **fuzzy affinity function** which we define as follows:

$$\psi_{aff}^{O_{f_i}}(p,q) = \begin{cases} \mu_{adj}^{O_{f_i}}(p,q) \cdot \mu_{app}^{O_{f_i}}(p,q) & if\ p \neq q \\ 1 & otherwise \end{cases}$$

$$\mu_{app}^{O_{f_i}}(p,q) = w_1 \cdot \Pr\left(x = \frac{(D_p + D_q)}{2}; \theta_1^{O_{f_i}}\right) + w_2 \cdot \Pr\left(x = |D_p - D_q|; \theta_2^{O_{f_i}}\right)$$

$$\mu_{adj}^{O_{f_i}}(p,q) = \begin{cases} 1 & if\ q \in N_p \\ 0 & otherwise \end{cases}$$

Fuzzy Location Prior - $\mu_{floc}^{O_{f_i}}(i)$

Given prior knowledge about the location of the object O_{f_i} any spatial fuzzy set representing the likelihood that the pixel i is located inside the object can be used to define $\mu_{floc}^{O_{f_i}}(i)$.

2nd-order Clique Potential - $V_{ij}(f_i, f_j \mid D)$

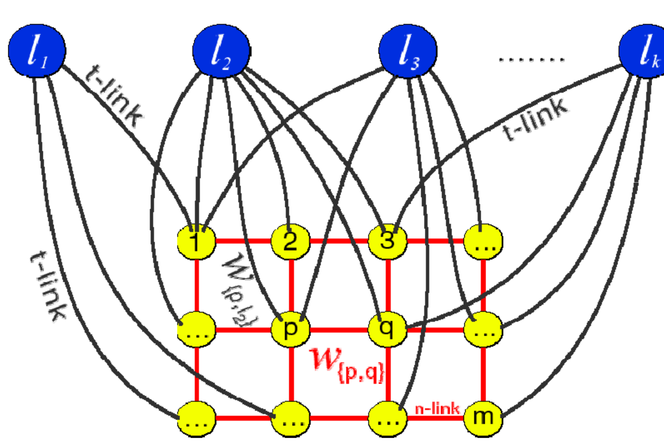
Measures the cost of jointly assigning a label f_i to the pixel i and a label f_j to its neighboring pixel $j \in N_i$ given any prior knowledge about the data D.

We model this using a Generalized Potts Interaction model as shown below:

$$V_{ij}(f_i, f_j \mid D) = K(i,j \mid D) \cdot \left(1 - \delta(|f_i - f_j|)\right) = \begin{cases} K(i,j \mid D) & if\ f_i \neq f_j \\ 0 & otherwise \end{cases}$$

$$K(i,j \mid D) = \exp\left(-(D_i - D_j)^T \Sigma_k^{-1} (D_i - D_j)\right)$$

Minimizing $E(f \mid D)$ using Graph-cuts



edge	weight	for
n-link - $\{i,j\}$	$K(i,j \mid D)$	$\{i,j\} \in N$
t-link - $\{i,l\}$	$W_i - V_i(l)$	$i \in P, l \in L$

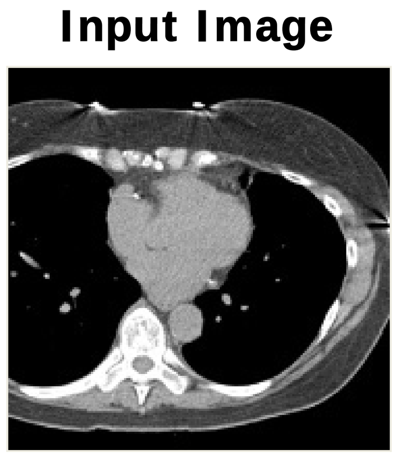
Minimizing $E(f \mid D)$ is equivalent to finding a minimum cost multi-way cut in the graph shown above.

For L = {0,1}, a global minimum can be obtained in polynomial time by solving the s-t mincut problem.

For |L| > 2, the optimal multi-way cut problem is NP-Hard. In such cases, good approximation algorithms are available.

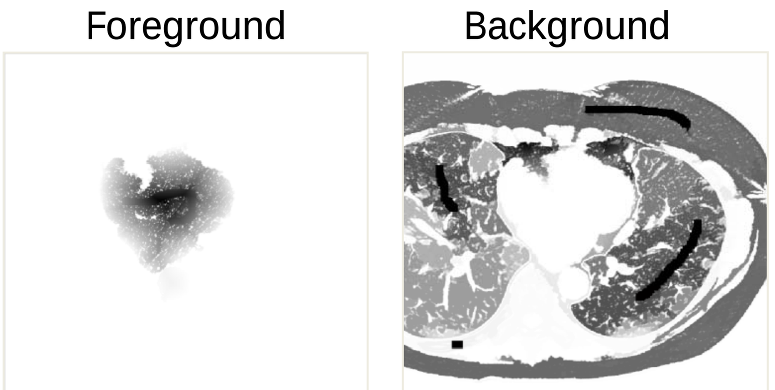
Application to 2D Heart Segmentation

Input Image



1st-order clique potentials

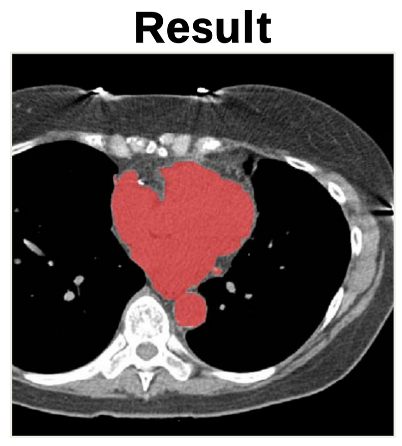
Foreground Background



2nd- order clique potentials

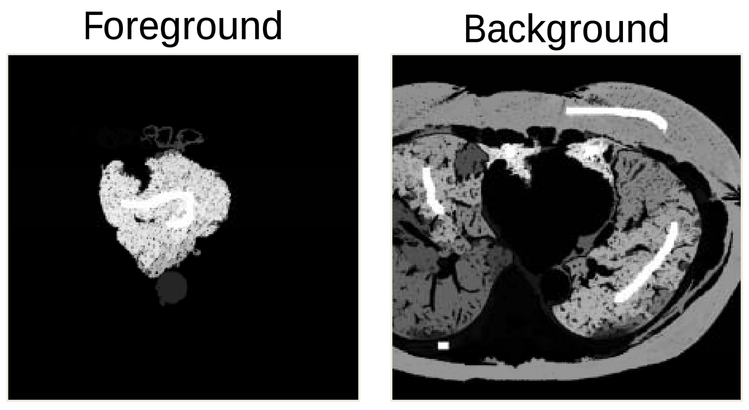
X-Dir Y-Dir

Segmentation Result



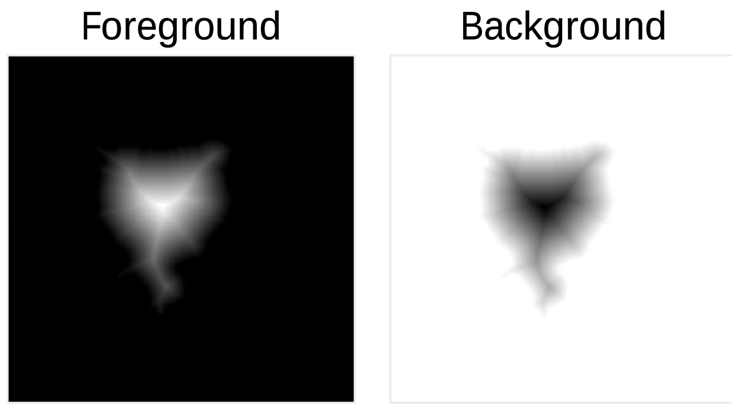
Fuzzy Connectivity Prior

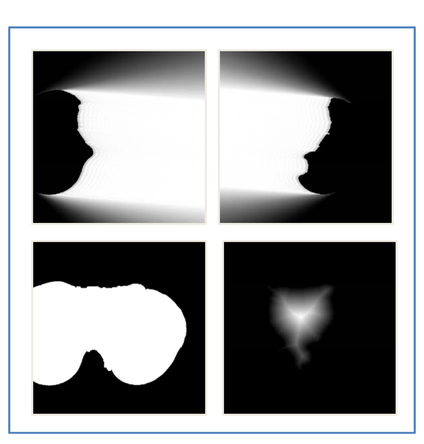
Foreground Background



Fuzzy Location Prior

Foreground Background





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Acknowledgements

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