Data-Driven Extrapolation of Plausible 3D Geometry

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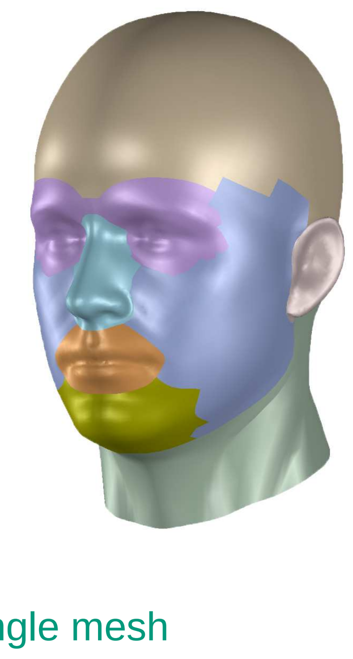
Background

Morphable shape models: Data-driven “shape space” models of geometry (and sometimes texture). Popular in (but not limited to) vision an graphics of human heads and faces, e.g. Vetter & Blanz, 1999.



Typical methodology:

- The domain is split up into <u>regions</u>, as a single PCA model can cover only limited variance of shape. Regions <u>overlap</u> to guarantee smooth borders.
- For each region, a PCA of its geometry is computed:

$\mathbf{b}_i = (x_1 y_1 z_1 \ldots x_V y_V z_V)^T$ — geometry of the region in a single mesh

$\mathbf{B} = [\mathbf{b}_1 - \boldsymbol{\mu}, \ldots, \mathbf{b}_N - \boldsymbol{\mu}],\ \boldsymbol{\mu} = N^{-1}\sum_{i=1}^{N} \mathbf{b}_i$ — mean centred region geometry of all meshes

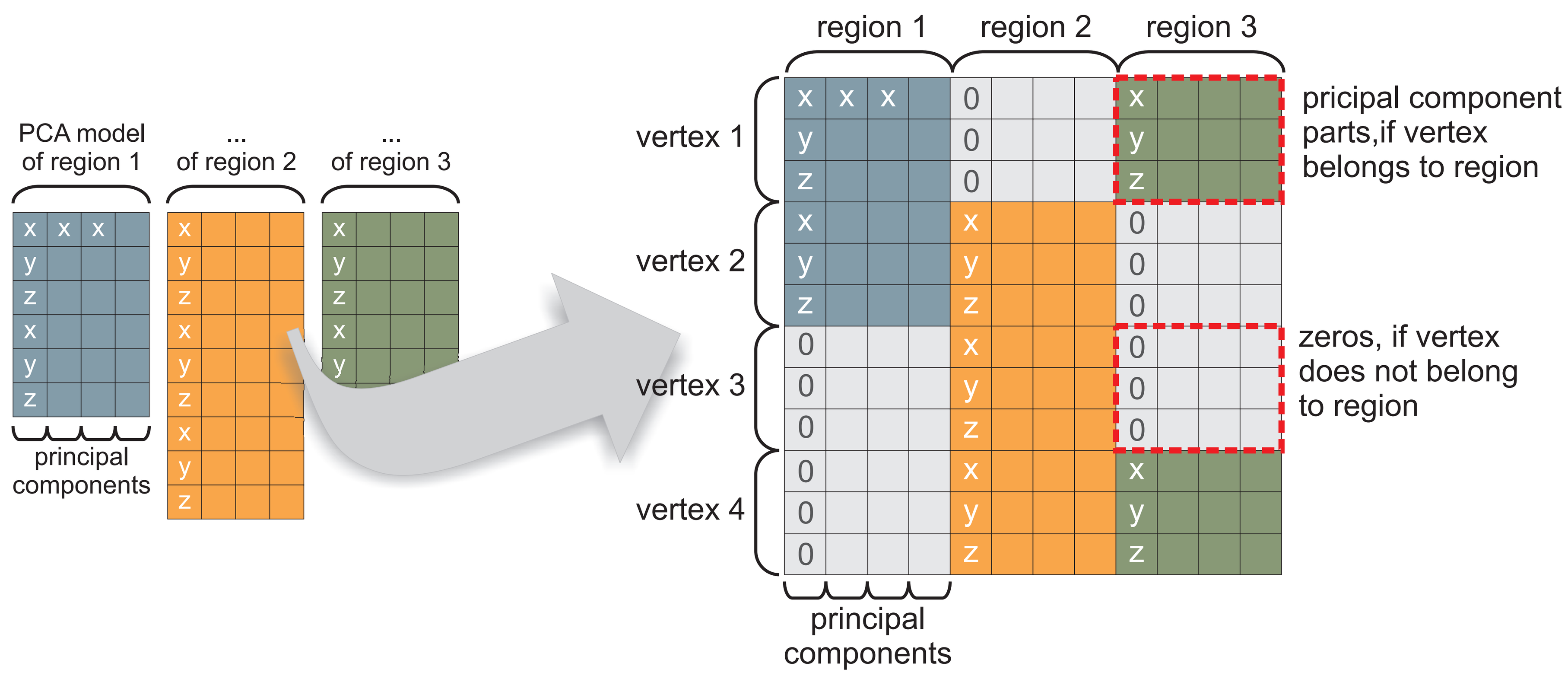
$\mathbf{P\Lambda P}^T = \mathbf{B}^T\mathbf{B},\ \mathbf{P}^T\mathbf{P} = \mathbf{I}$ — spectral decomposition yields PC-matrix

- Where regions overlap, locations of vertices are determined by multiple PCA models that have to be weighted (”PCA mixture model”).

Compact linear formulation of PCA mixture model

Multiple PCA models are awkward to compute with. However, a <u>compact linear formulation</u> is possible.

- Construction of the <u>combined model matrix</u> $\mathbf{M}$, subsuming all region PCA models, as follows:



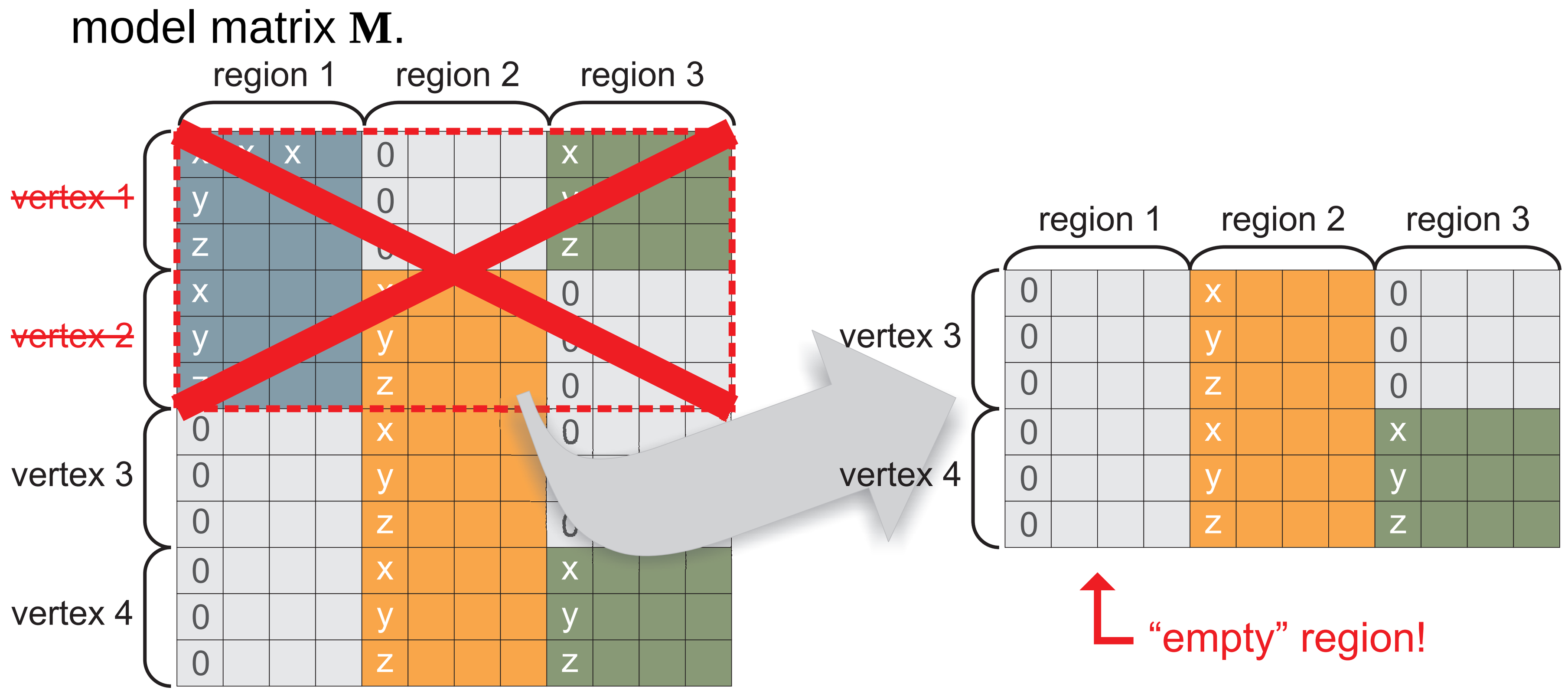
- Not shown in schematic: Weighting of matrix blocks where the regions overlap. Different weighting schemes possible.

Data-driven extrapolation

Problem statement: Given an <u>arbitrarily small set of vertices</u>—e.g. a patch, the profile curve of a face or some landmarks—construct a plausible full shape (here: a head) fitting the given geometry.

Generic extrapolation strategy for linear models (not limited to PCA):

- Eliminate all rows corresponding with unknown vertices from combined model matrix $\mathbf{M}$.



- Solve resulting system for shape space parameters.
- Use parameters with full combined model matrix to extrapolate the compete shape:

SVD of reduced model — given geometry

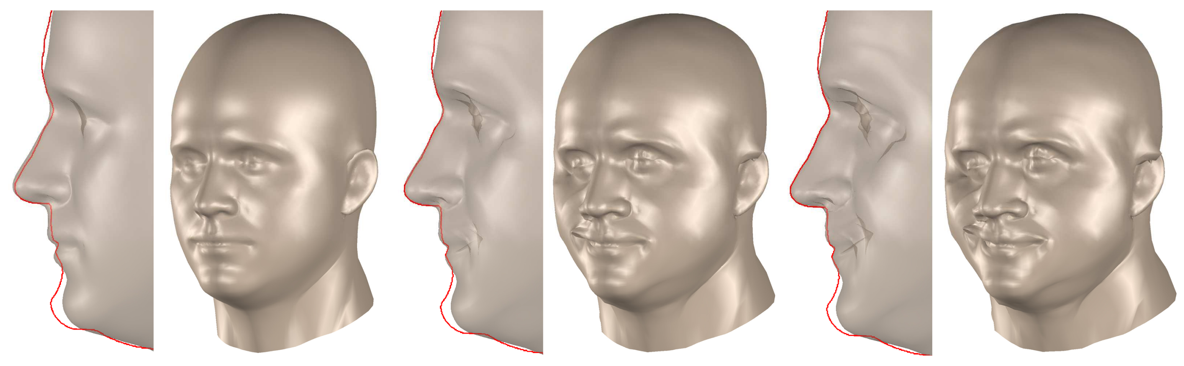
shape parameters → $\mathbf{m} = \mathbf{M}_{red}^{+}(\mathbf{g}_{red} - \boldsymbol{\mu})$

extrapolated geometry → $\mathbf{g}_{full} = \mathbf{M}_{full}\mathbf{m} + \boldsymbol{\mu}$

complete model

Problems

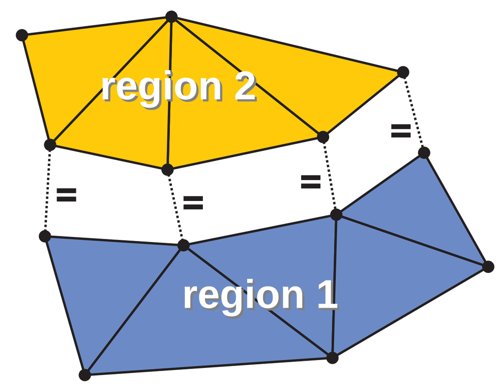
- Reduced system is underdetermined if regions without known geometry exist (zero-blocks in model matrix).
- Overfitting to known geometry leads to distorted extrapolation result.
- Common regularization strategy—limiting the norm of the shape parameter vector—only increases fitting error.



Fitting errors due to regularization of shape parameter norm, distortion due to overfitting.

<u>A better regularization is required!</u>

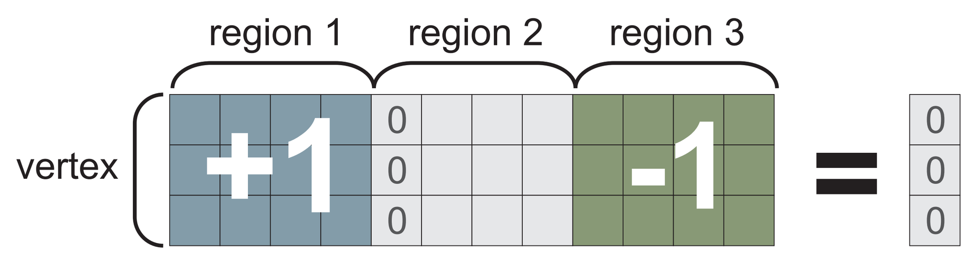
Regularization with “smooth mesh prior”



Idea: In regions without known geometry, models should yield the same location for shared vertices. This can be enforced by <u>Tikhonov regularization</u>.

Therefore:

- Identify vertices belonging to two regions (vertices belonging to more than two regions ignored for simplicity).
- These vertices have three rows with two non-zero blocks in the combined model matrix.
- Multiplying the blocks with +1 and –1, Each of these vertices yields three lines in the Tikhonov matrix $\boldsymbol{\Gamma}$:



- Extrapolation is then computed as:

$$\mathbf{m} = \begin{bmatrix} \mathbf{M}_{red} \\ \lambda\boldsymbol{\Gamma} \end{bmatrix}^{+} \begin{bmatrix} \mathbf{g}_{red} - \boldsymbol{\mu} \\ \mathbf{0} \end{bmatrix}$$

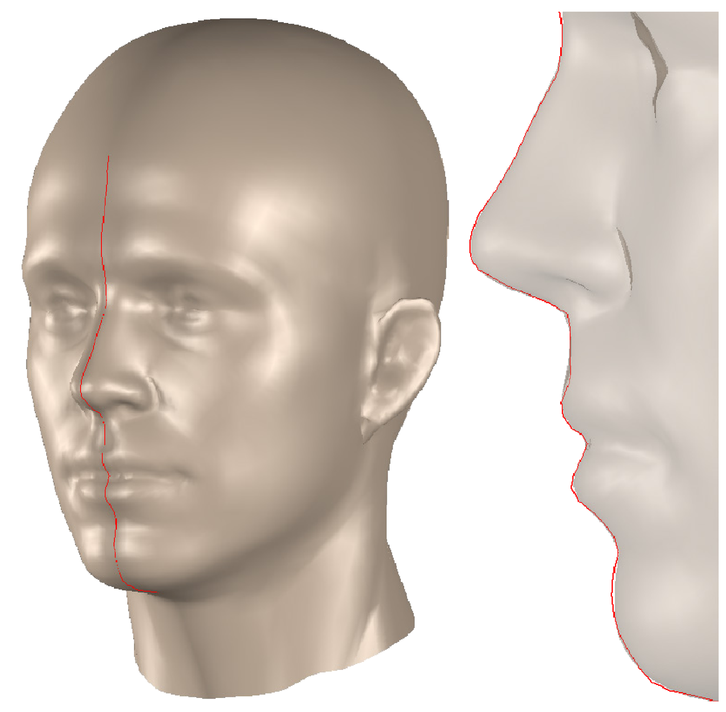
weighted regularization

$$\mathbf{g}_{full} = \mathbf{M}_{full}\mathbf{m} + \boldsymbol{\mu}$$

Results

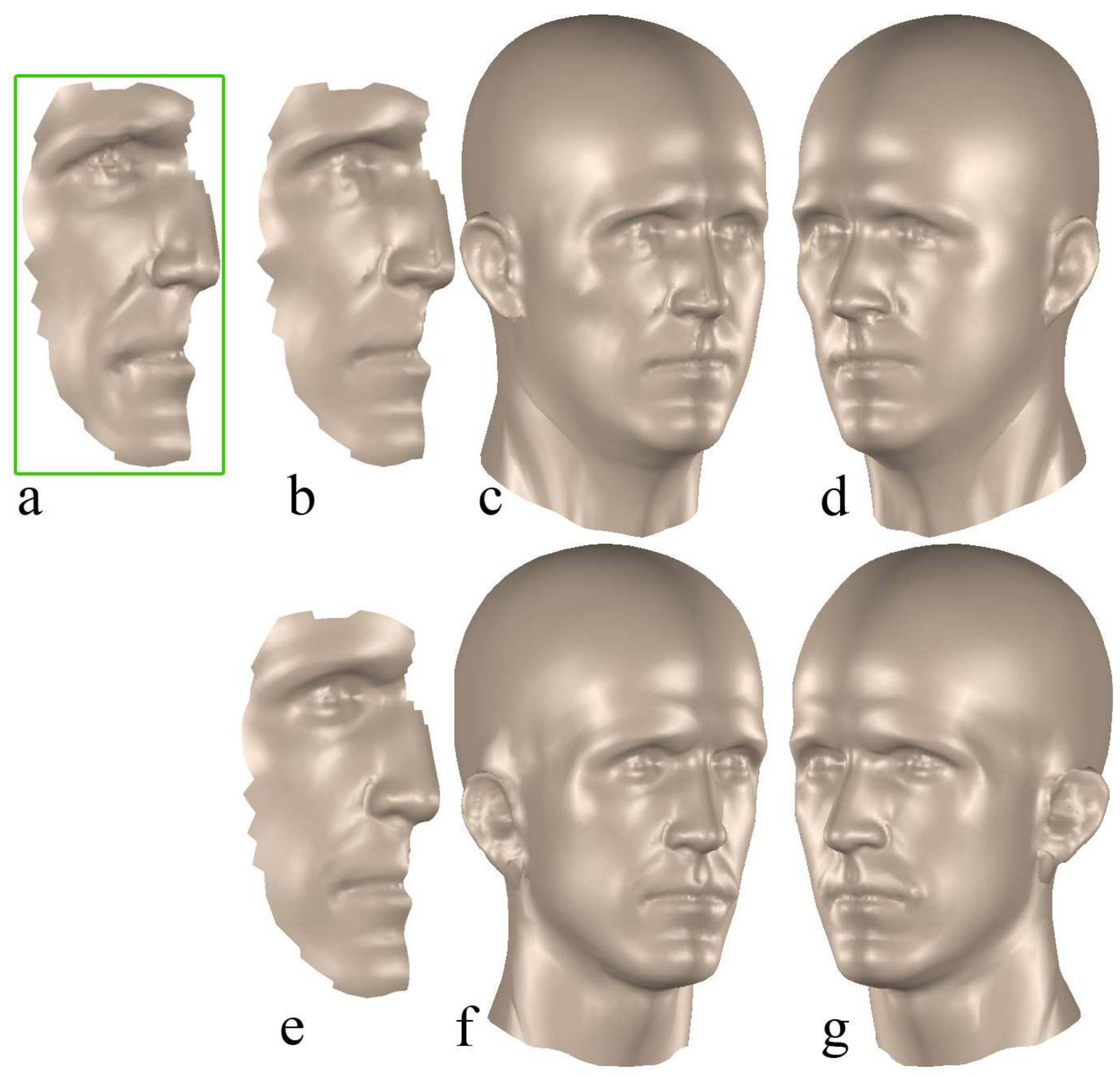
Extrapolation from a profile curve.

The transparent view shows the precision of the fitting to the given profile curve (red).



Extrapolation from a small patch.

(a) given geometry
(b) reconstruction of (a) by the model
(c,d) extrapolation of complete head from (a), right and left view
(e) reconstruction of (a) by a model with symmetry constraints
(f,g) extrapolation of complete head from (a) by a model with symmetry constraints, right and left view



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