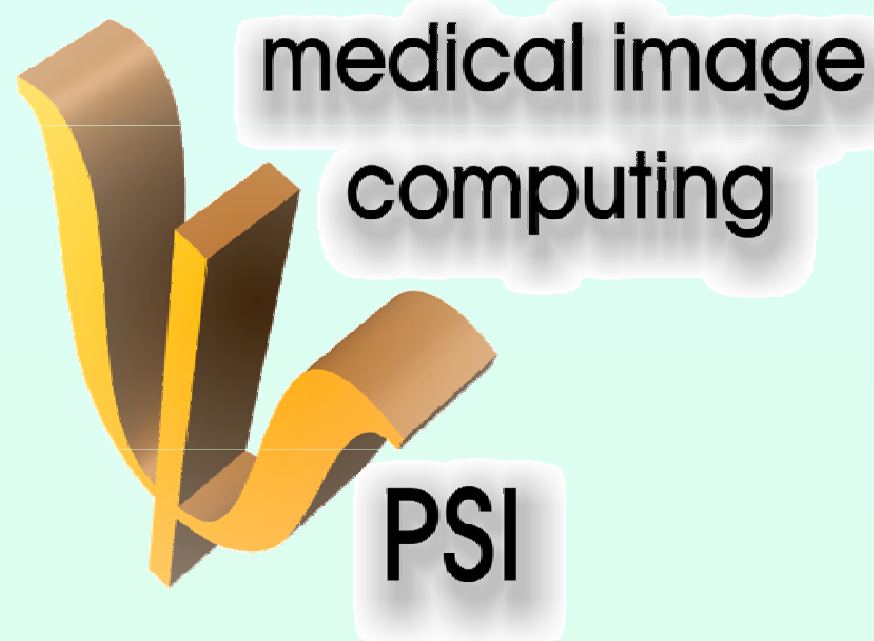




# The Isometric Deformation Model for Object Recognition



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## Abstract

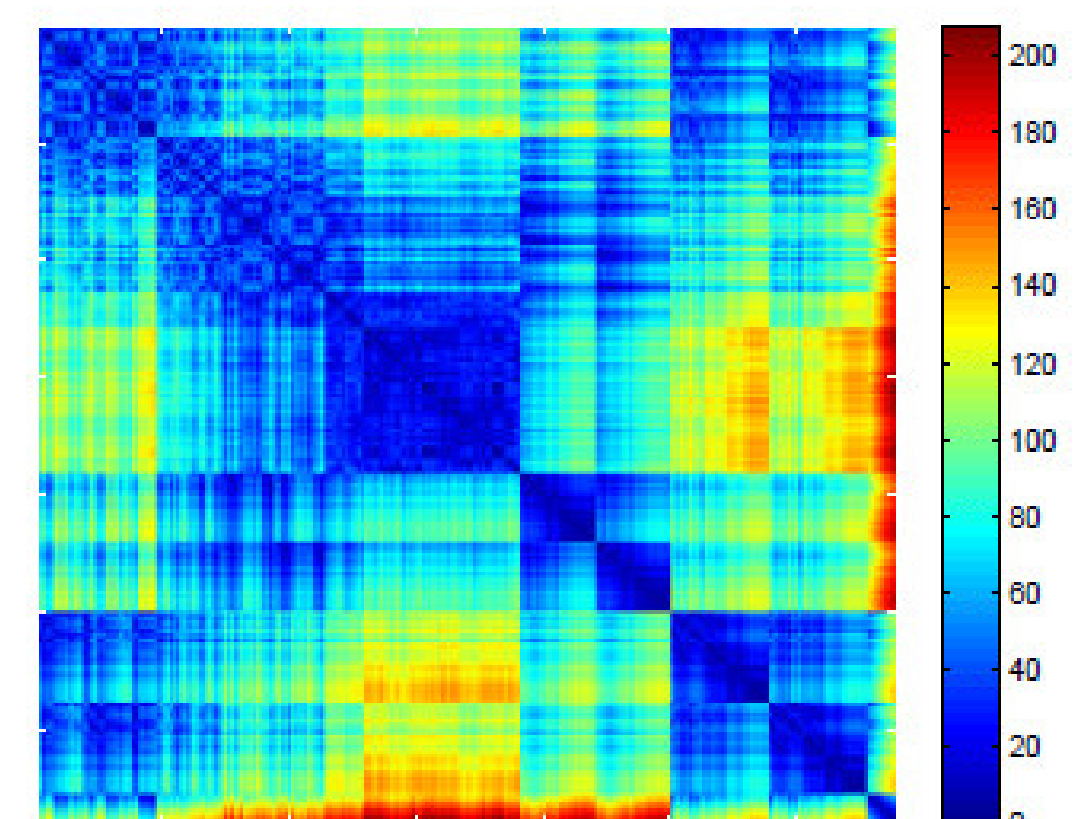
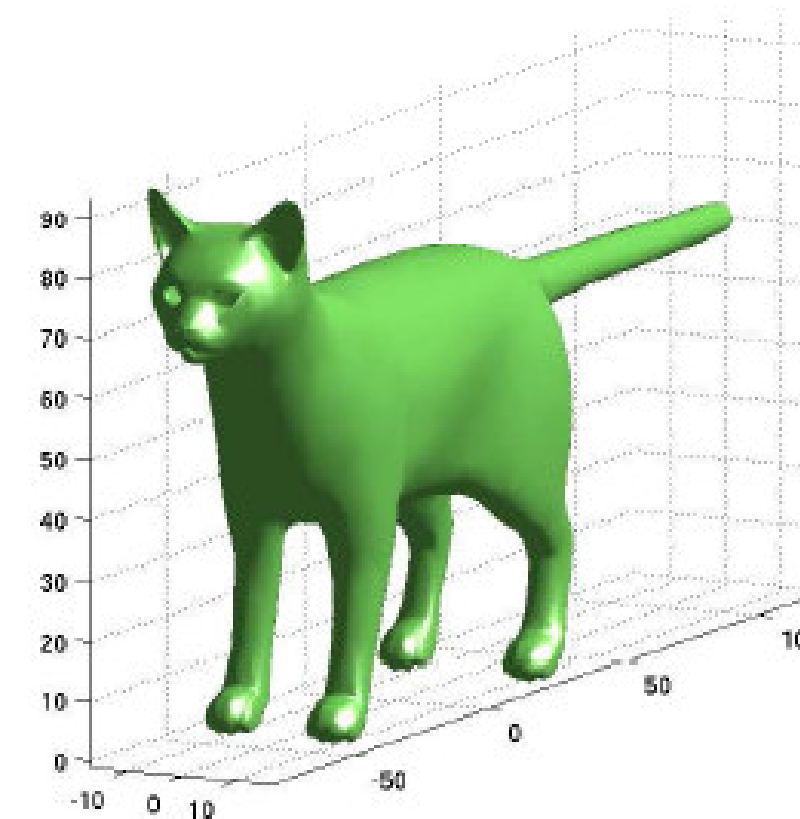
Different methods for object recognition using the isometric deformation model are presented. The methods are built upon the use of geodesic distance matrices (GDM) as an object representation. The first method compares these matrices by using histogram comparisons. The second method is a modal approach. The largest singular values or eigenvalues appear to be an excellent shape descriptor, based on the comparison with other methods also using the isometric deformation model and a general baseline algorithm. The methods are validated using the TOSCA database of non-rigid objects and a rank 1 recognition rate of 100% is reported for the modal representation method using the 50 largest eigenvalues. This is clearly higher than other methods using an isometric deformation model.

## Object representation

Objects are represented as geodesic distance matrices (GDM), because they are invariant for isometries (distance preserving transformations). We call  $G$  a GDM if  $G = [g_{ij}]$ , with  $g_{ij}$  the geodesic distance\*\* between points  $i$  and  $j$ . However, given the object, the GDM is determined up to a simultaneous permutation of rows and columns. Also variants of the GDM are used:

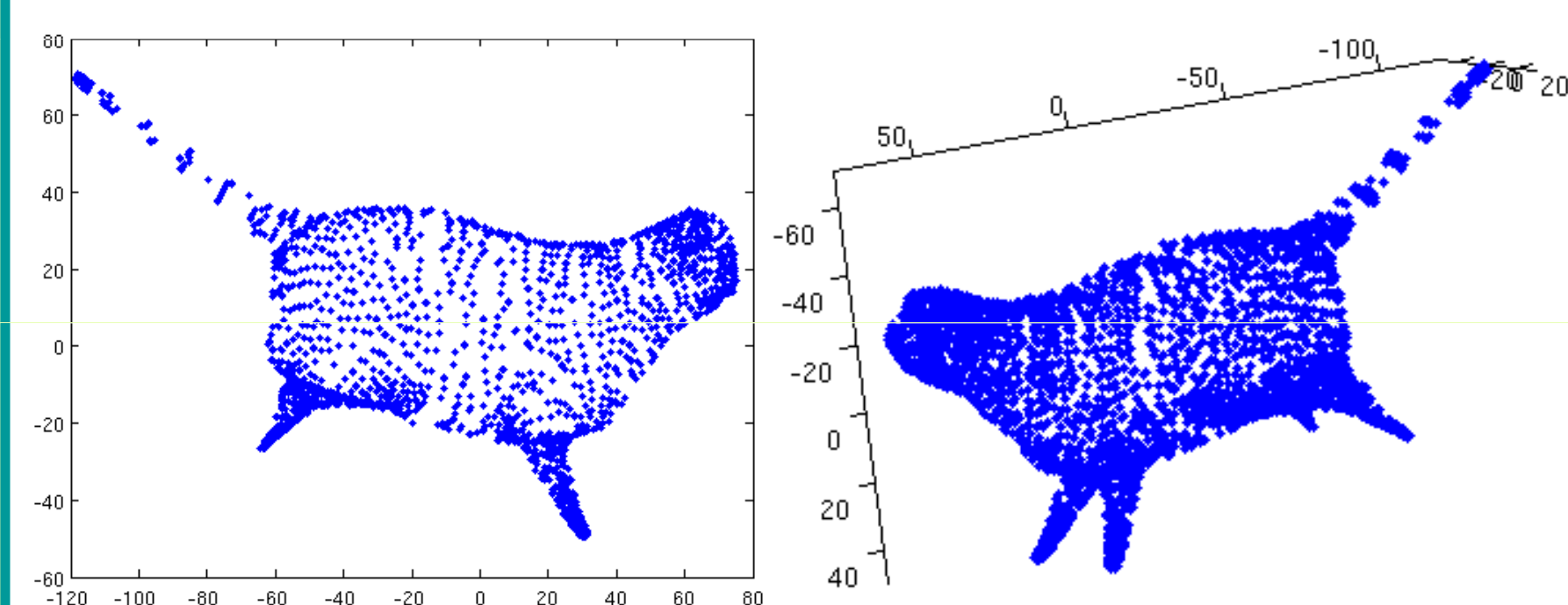
$$\begin{aligned} G_1 &= [g_{ij}] \\ G_2 &= [g_{ij}^2] \\ G_3 &= [\exp(-g_{ij}^2 / (2\sigma^2))] \\ G_4 &= [1 + \frac{1}{\sigma} g_{ij}]^{-1} \end{aligned}$$

\*\*The geodesic distance between two points is the length of the shortest path on the object surface between two points on the object.



## Multidimensional scaling

Multidimensional scaling (MDS) is a technique that allows visualization of the proximity between points with respect to some kind of dissimilarity (distance) measure matrix. Using a GDM, MDS provides a canonical form in an arbitrary dimension (left: 2D, right: 3D) [1].



Math:

$$B_{\Delta} = -\frac{1}{2} J \Delta J$$

$$B_{\Delta} = V \Lambda V^T$$

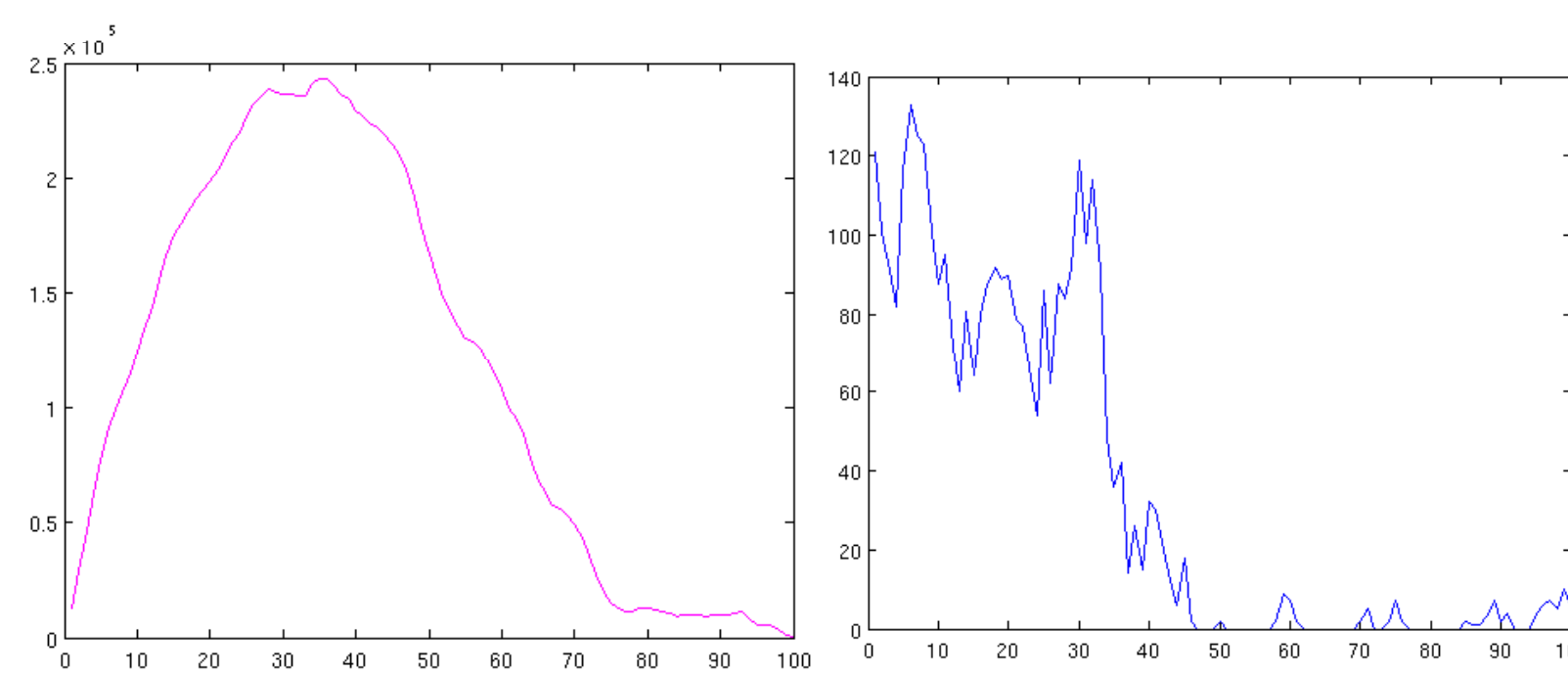
$$X' = V_+^m (\Lambda_+^m)^{1/2}$$

Comparison:  
Registration error

[1] A.M. Bronstein, M.M. Bronstein, R. Kimmel, "Expression-invariant 3D face recognitions", AVBPA, 2003, p 193-204.

## Histogram comparison

Histograms of the values of the GDM are invariant for the matrix permutations and are therefore suitable shape descriptors. We considered the histogram of all values (left) using 100 bins and the histogram of the pointwise averaged values (right) with 80 bins.



Comparison:  
See below: dissimilarity measures

## Modal representation

In the third approach, the information in the geodesic distance matrix is separated into a matrix that contains intrinsic shape information and a matrix with information about corresponding points. This is done with an eigenvalue decomposition (EVD) or a singular value decomposition (SVD) of the GDM.

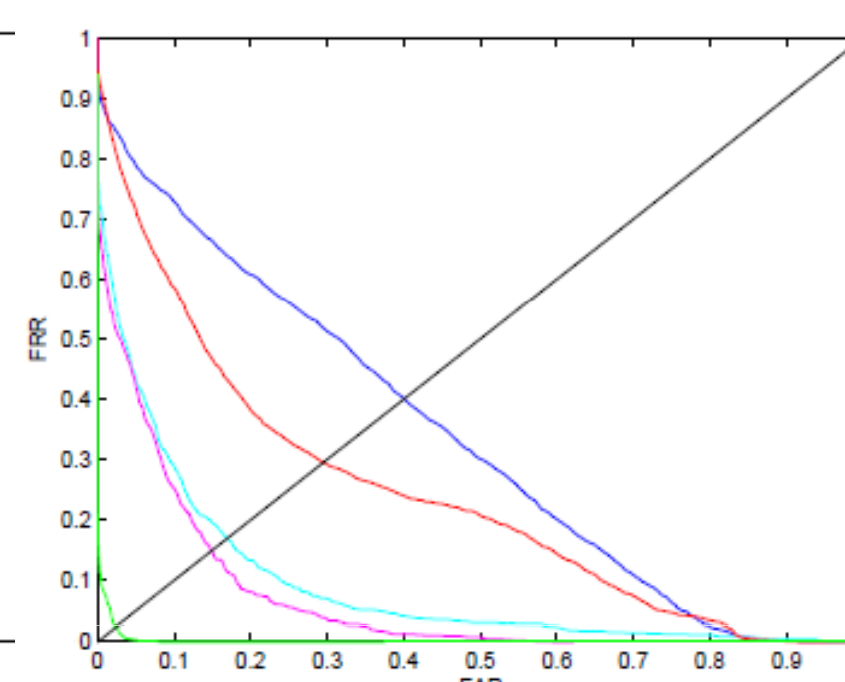
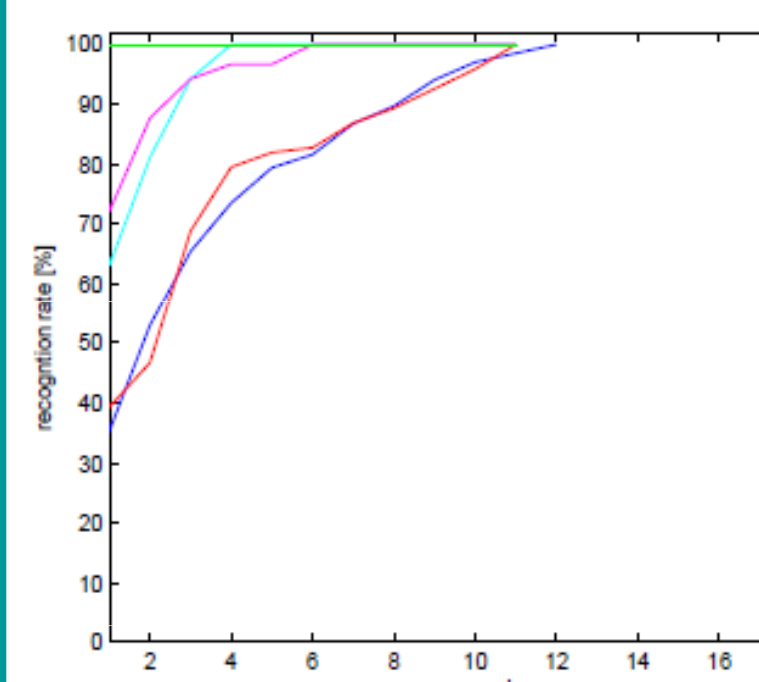
The 50 largest eigenvalues seem to be an excellent shape descriptor.

## Dissimilarity measures

Dissimilarity measure	Formula
Jensen-Shannon Divergence	$D_1 = H(\frac{1}{2}S^k + \frac{1}{2}S^l) - (\frac{1}{2}H(S^k) + \frac{1}{2}H(S^l))$
Mean normalized Manhattan distance	$D_2 = \sum_{i=1}^m \frac{2 S_i^k - S_i^l }{S_i^k + S_i^l}$
Mean normalized maximum norm	$D_3 = \max_i \frac{2 S_i^k - S_i^l }{S_i^k + S_i^l}$
Mean normalized absolute difference of square root vectors	$D_4 = \sum_{i=1}^m \frac{2 \sqrt{S_i^k} - \sqrt{S_i^l} }{\sqrt{S_i^k} + \sqrt{S_i^l}}$
Correlation	$D_5 = 1 - \frac{S^k \cdot S^l}{\ S^k\  \ S^l\ }$
Euclidean distance	$D_6 = \sqrt{\sum_{i=1}^m (S_i^k - S_i^l)^2}$
Normalized Euclidean distance	$D_7 = \sqrt{\sum_{i=1}^m (S_i^k - S_i^l)^2 / \sigma_i^2}$
Mahalanobis distance	$D_8 = \sqrt{\sum_{i=1}^m (S^k - S^l)^T \text{cov}(S)^{-1} (S^k - S^l)}$

## Experimental results

Results of standard recognition tests (CMC/ROC) on the TOSCA database containing 133 non rigid deformed objects of 9 subjects. Object recognition with a baseline algorithm (blue) is compared to object recognition using MDS (red), histogram comparison of PWA (cyan) and all values (magenta) and modal representation (green).

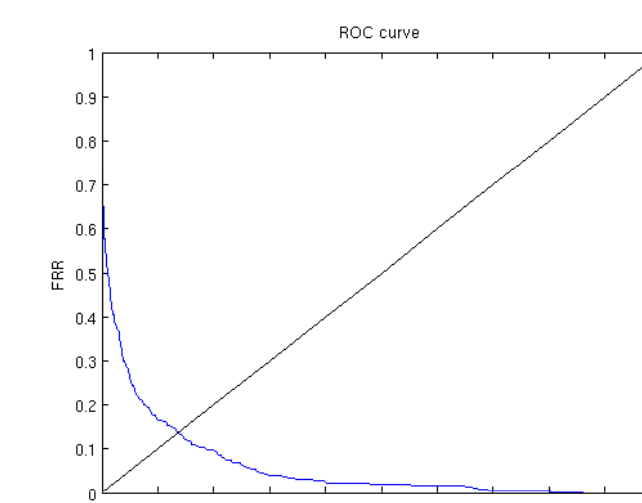
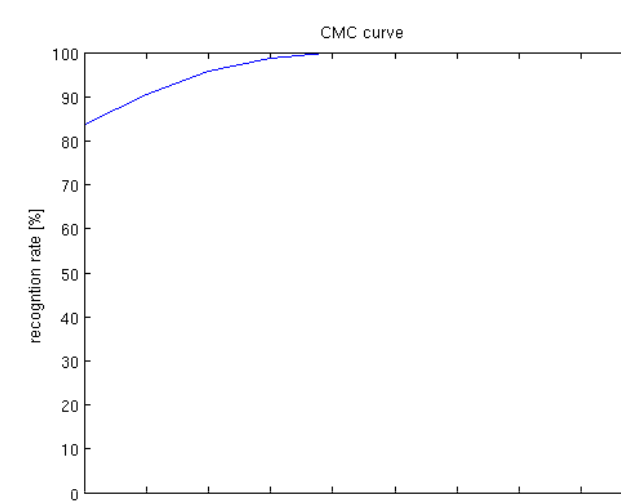


Diss measure	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	$D_7$	$D_8$
PWA value Histogram comparison								
$R_1RR$	45.08%	54.92%	45.08%	54.92%	46.72%	56.56%	63.11%	20.49%
$EER$	18.68%	15.83%	25.31%	15.69%	34.68%	23.13%	16.93%	42.07%
All value Histogram comparison								
$R_1RR$	67.21%	69.67%	47.54%	69.67%	58.20%	72.13%	66.39%	20.49%
$EER$	14.95%	15.26%	21.01%	15.26%	19.63%	14.90%	16.94%	48.37%
Modal representation								
$R_1RR$	84.43%	100.0%	85.25%	100.0%	54.92%	76.23%	97.54%	33.79%
$EER$	10.11%	2.43%	10.09%	2.44%	20.33%	10.74%	7.74%	34.18%

Also we did the experiments with some variants of the geodesic distance matrix.

	$G_1$	$G_2$	$G_3$	$G_4$
All Value Histogram comparison				
$R_1RR$	72.13%	70.49%	70.49%	69.67%
$EER$	14.90%	14.01%	14.14%	15.42%
Modal representation				
$R_1RR$	100.0%	97.54%	71.31%	90.98%
$EER$	2.43%	3.47%	17.79%	12.25%

Preliminary results for face recognition on BU3D-FE database containing



88 faces from 10 subjects. This gives an rank 1 recognition rate of  $\pm 84\%$ .

[2] D. Smeets, T. Fabry, J. Hermans, D. Vandermeulen, P. Suetens, "Isometric deformation modelling for object recognition", CAIP, 2009 (submitted).