RIEMANNIAN PERONA-MALIK DIFFUSION FOR ORIENTATION DISTRIBUTION FUNCTION IMAGES

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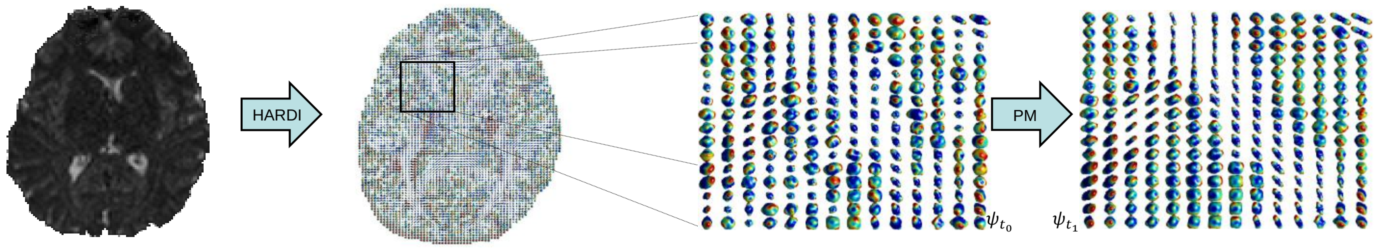
Abstract

We generalize the Perona-Malik diffusion equation to ODF Images within a Riemannian framework. To this end, we derive the PM diffusion equation from an energy functional. Discretization as well as a numerical update scheme for solving resulting initial value problems are developed.
We provide a stability analysis of our update scheme and propose an effective implementation by means of spherical harmonics. We demonstrate the performance of our approach on synthetic as well as real data.

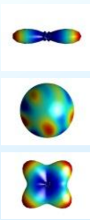
Results

- Isotropic nonlinear diffusion for HARDI by respecting Riemannian structure.
- Discrete forms of first and second order derivatives for HARDI.
- Numerical stable diffusion scheme as long as $\|\delta t \delta E\| < \infty$ in contrast to Euclidean case.
- Developing ODFs in series of spherical harmonics reducing computational costs significantly.
- Experiments show visually and quantitatively better noise reduction capabilities than previous approaches.



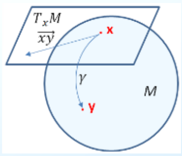
Introduction

- PM diffusion methods are known for i.e. vector-valued image data, but is new for High Angular Resolution Diffusion Images (HARDI)
- HARDI encode in every pixel an ODF and is an extension of Diffusion Tensor MRI measuring water self-diffusion in 3D
- The main application of HARDI is in biological and medical applications for in vivo imaging of tissues,



Theoretical backround

General Riemannian framework of Goh et al. [1]



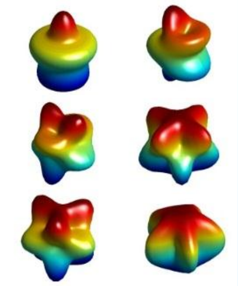
- Consideration of Riemannian manifolds and related definitions
- handsome treatment of ODFs with square root representation
- Geodesic marching scheme: $\psi^{t+1} = \exp_{\psi^t}(-\delta t \delta E)$

Own contribution

- Deriving Perona Malik Diffusion from energy

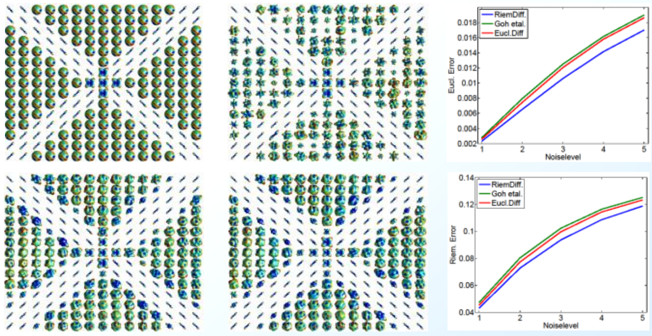
$$E = \frac{1}{2}\int_{\Lambda} \phi(\|\nabla\psi\|_{\psi}^{2})dx$$

- Discrete formulation with Taylorexpansion $\psi(x_i \pm \epsilon e_j) = \psi_i \pm \epsilon\partial_j\psi_i + \frac{\epsilon^2}{2}\partial_j^2\psi_i \pm O(\epsilon^3)$ to derive first and second order spatial derivations needed for marching scheme



- numerical stable diffusion scheme because of Riemannian structure
- Spherical harmonics representing functions on the unit sphere, reduces the computational costs significantly.

Experiments



Left: Synthetic crossing fibers experiment: Original image, noisy image, Riem. Isotr. diffusion and eucl. Isotr. diffusion on noisy image.
Right: Riemannian (top) and Euclidean (bottom) error versus different noise levels.

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[1] Goh, A. et al.: A nonparametric Riemannian framework for processing high angular resolution diffusion images (hardi). CVPR (2009)