SHAPE COULOMBIZATION

Boscaini D., Girdziusas R. and Bronstein M. M. – University of Lugano
davide.boscaini@usi.ch

ICVSS 2014
Sicily ~ 13-19 July
International Computer Vision Summer School

Abstract

Canonical shape analysis is a popular method in deformable shape matching, trying to bring the shape into a canonical form that undoes its non-rigid deformations, thus reducing the problem of non-rigid matching into a rigid one.

As a result, the shape canonization process replaces the original shape by its stretched-the-most variant.

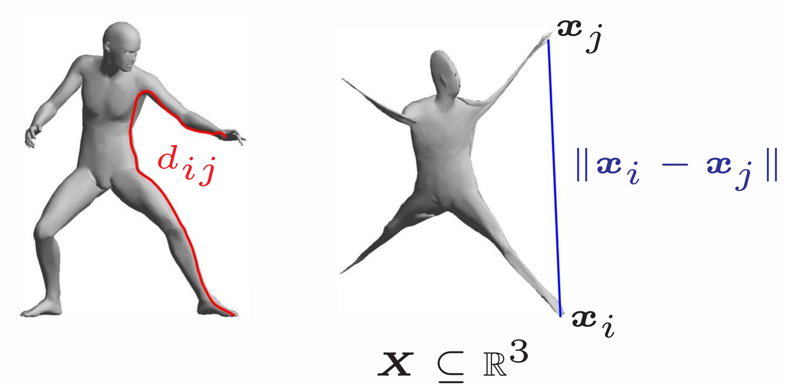
Inspired by natural phenomena, we propose to perform such a stretching by the simulation of electrostatic repulsion among the vertices of the shape.

Related works

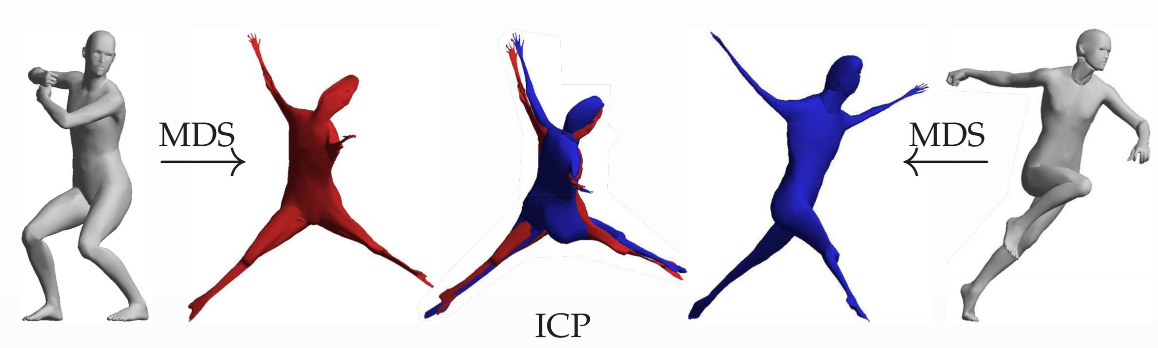
Elad and Kimmel [3] proposed to perform the canonization by measuring **geodesic distances** on the shape and embedding them into a Euclidean space by means of **multidimensional scaling** (MDS):

$$\min_{\boldsymbol{X}=\{\boldsymbol{x}_1,\ldots,\boldsymbol{x}_n\}} \sum_{i,j=1}^{n} (d_{ij} - \|\boldsymbol{x}_i - \boldsymbol{x}_j\|)^2,$$

where d_{ij} = intrinsic metric.

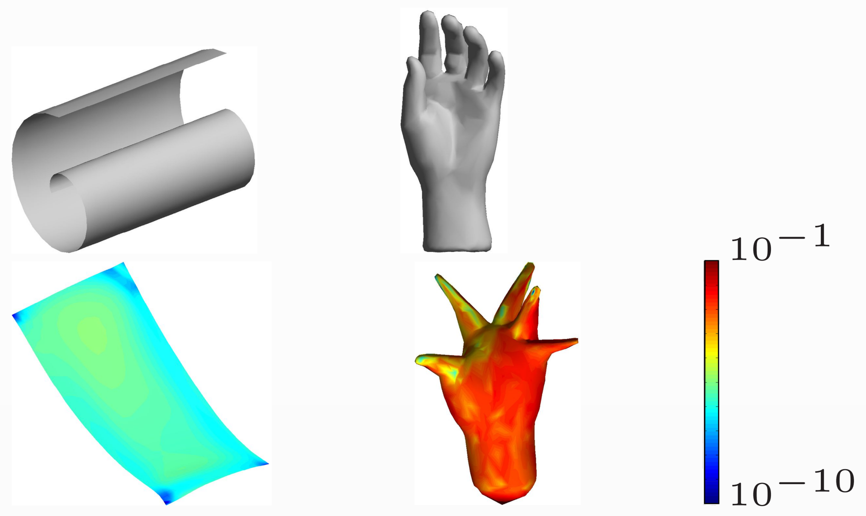


If the embedding is **isometric**, then **intrinsic** similarity between original shapes = **extrinsic** similarity between canonical forms:

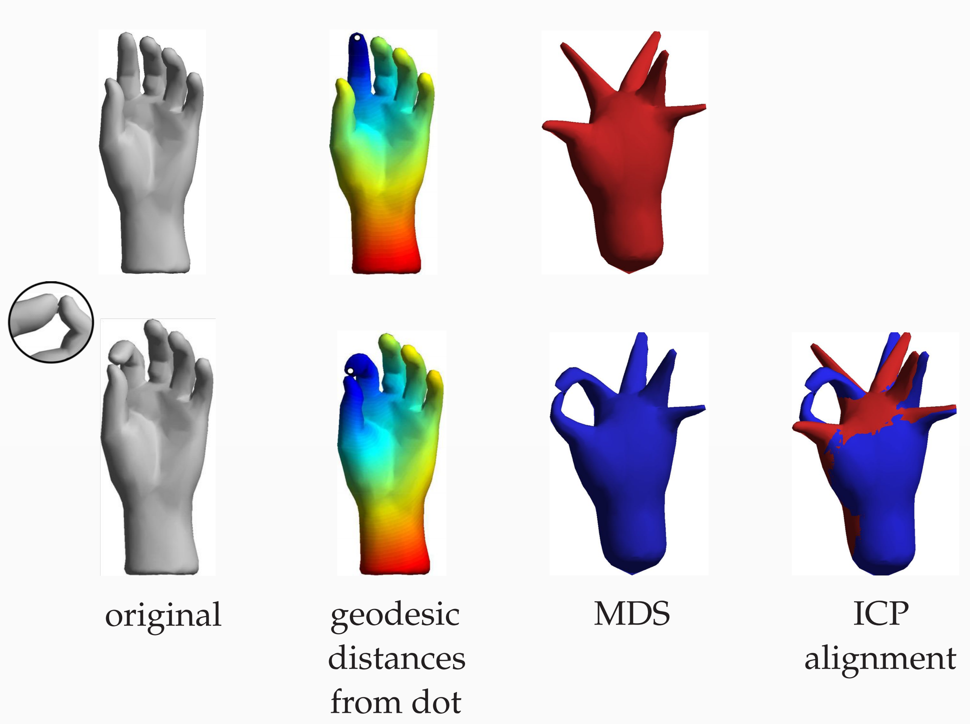


Main drawbacks:

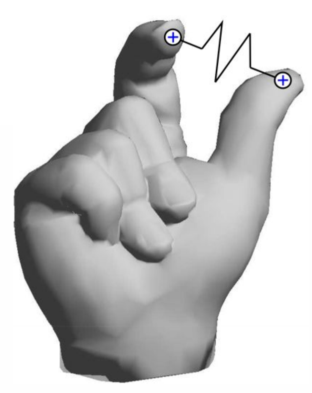
- distortion is data dependent



- sensitivity to topological noise



Our approach [5]



repulsion: $F \propto \frac{1}{\|\boldsymbol{x}_i - \boldsymbol{x}_j\|^2}$

metric constraints on edges $(i,j) \in E$ only:

$\|\boldsymbol{x}_i - \boldsymbol{x}_j\| = d_{ij}$

shape "Coulombization":

$$\min_{\boldsymbol{X}} \underbrace{\sum_{i \neq j} \frac{1}{\|\boldsymbol{x}_i - \boldsymbol{x}_j\|}}_{\text{Coulomb energy: } \mathcal{E}(\boldsymbol{X}) = -\nabla F} \quad \text{s.t. } \|\boldsymbol{x}_i - \boldsymbol{x}_j\| = d_{ij}$$

We propose to solve our problem using alternating minimization:

- step(s) of unconstrained minimization:

$$\boldsymbol{X}^{(t)} = \boldsymbol{X}^{(t-1)} - c\nabla\mathcal{E}(\boldsymbol{X}^{(t-1)})$$

- projection on metric constraints:

$$\boldsymbol{X}^{(t)} = \text{proj}(\boldsymbol{X}^{(t)})$$

If the metric constraints are imposed exactly, such a canonical representation is isometric (no metric distortion). However, since closed polyhedral surfaces are known to be **rigid**, it is necessary to relax the metric constraints.

How to overcome this difficulty? In [5] we propose to change the optimization using **approximate projections** on the constraints:

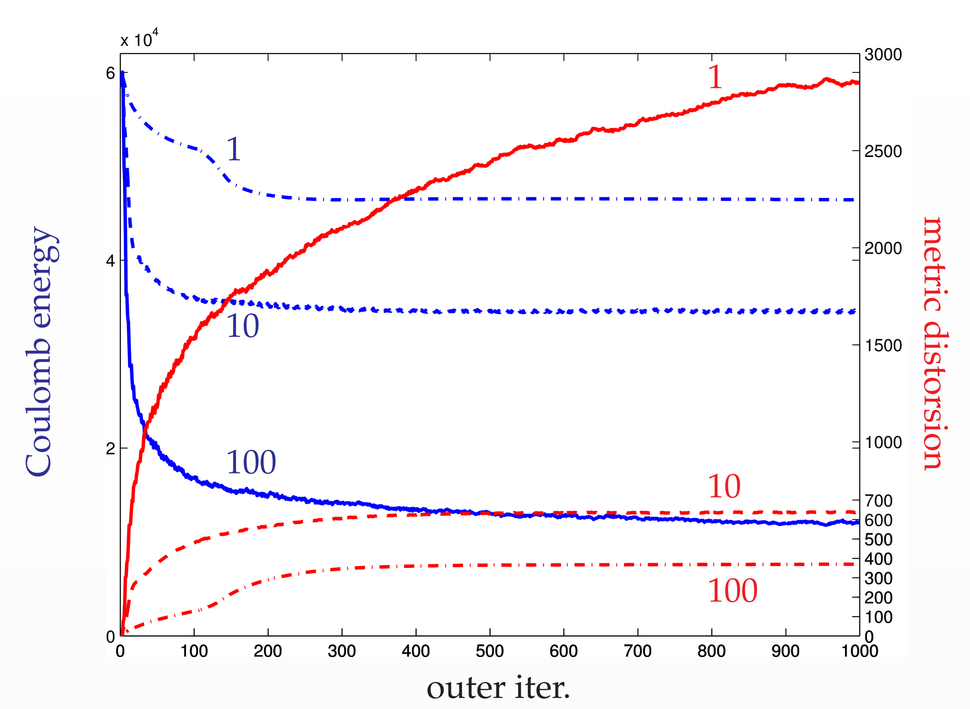
$$\sum_{(i,j)\in E} (d_{ij} - \|\boldsymbol{x}_i - \boldsymbol{x}_j\|)^2.$$

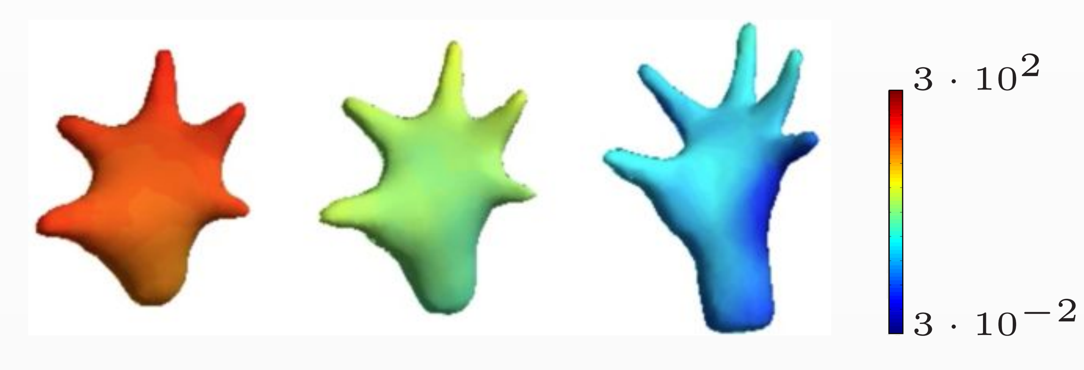
Groginsky [1] observes that the violation of the constraints can be minimized through the **fixed-point iteration**

$$\boldsymbol{x}_i^{(t+1)} = \frac{1}{\nu} \sum_{(i,j)\in E} \left(\boldsymbol{x}_j^{(t)} + d_{ij} \frac{\boldsymbol{x}_i^{(t)} - \boldsymbol{x}_j^{(t)}}{\|\boldsymbol{x}_i^{(t)} - \boldsymbol{x}_j^{(t)}\|} \right),$$

where ν stands for the valence of the vertex $\boldsymbol{x}_i$.

Distortion control



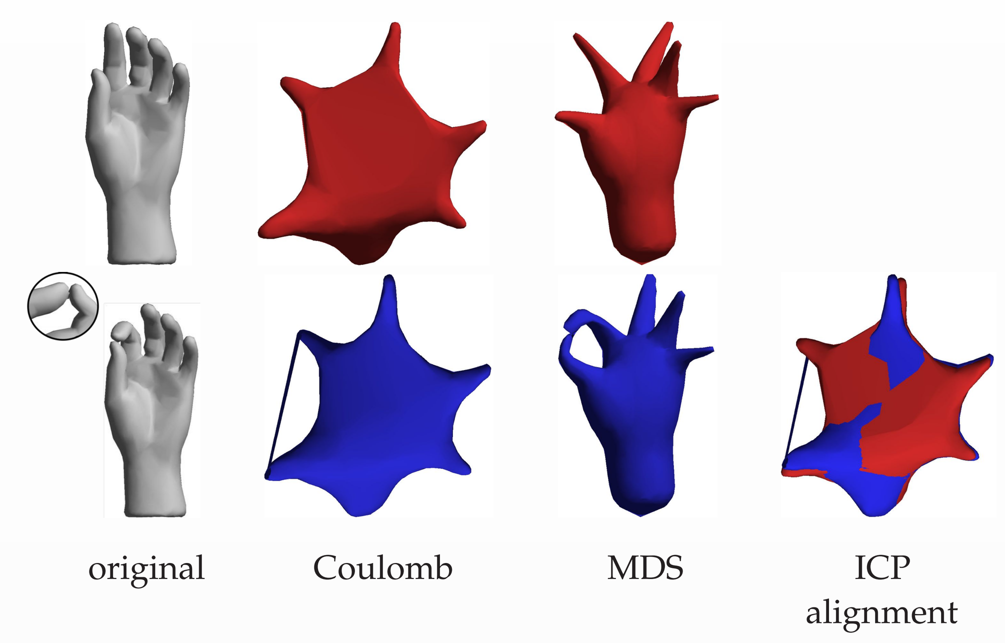


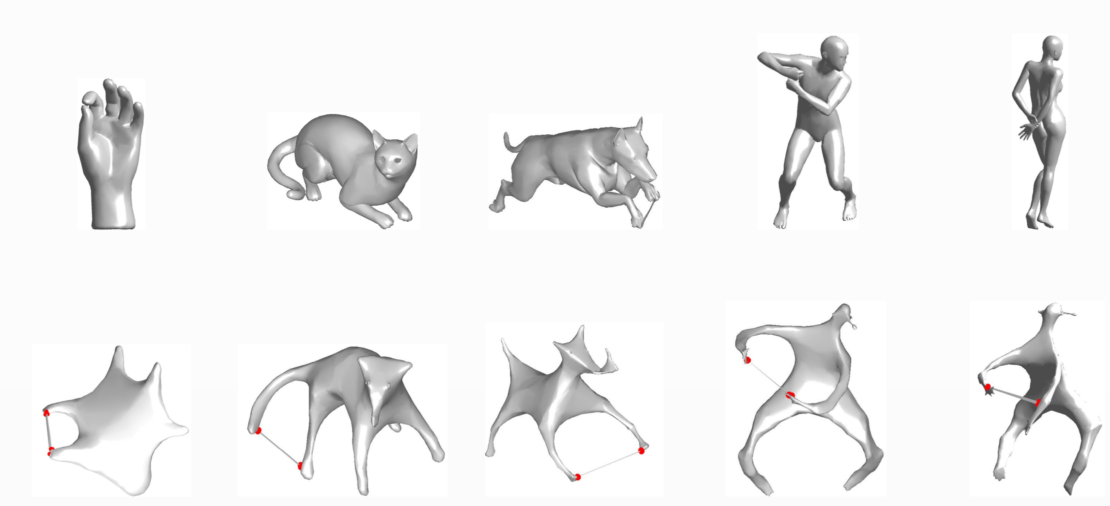
Handling topological noise

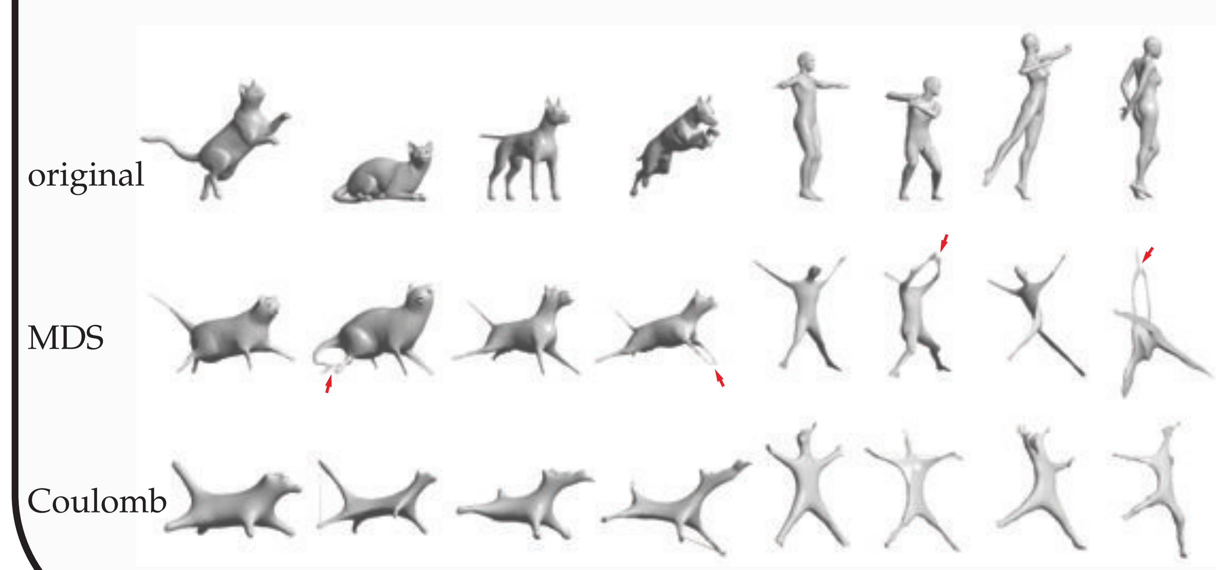
Due to the local nature of the topological noise, we consider an L^1 violation of the constraints

$$\sum_{(i,j)\in E} |d_{ij} - \|x_i - x_j\||,$$

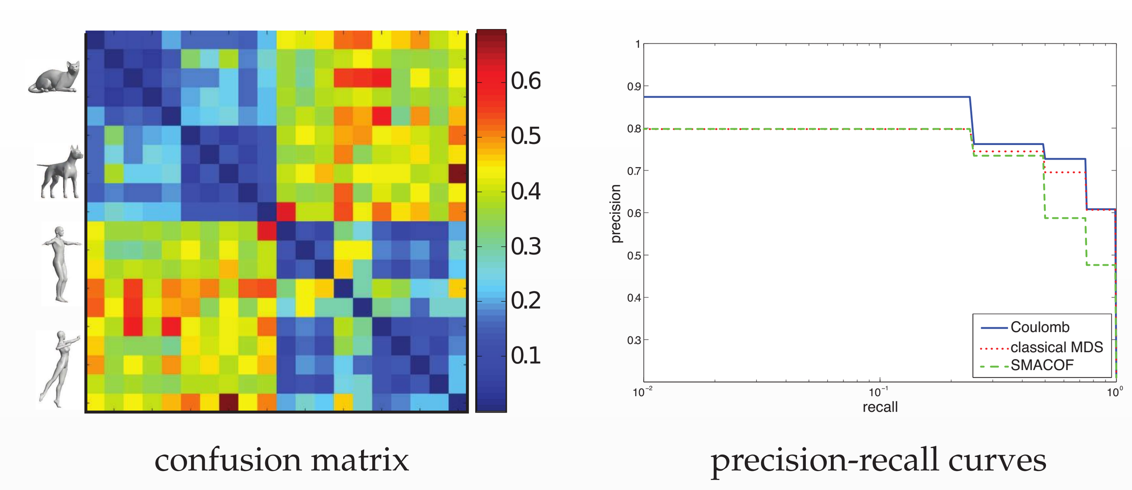
in order to exploit the sparsity-inducing properties of the L^1 norm. In [4], the authors shows that the new problem can be solved by a simple re-weighting of the previous fixed-point iteration.







Retrieval results



Shapes from TOSCA dataset.

References

[1] H.L. Groginsky, Position estimation using only multiple simultaneous range measurements, in *Aerosp. Navig. Electron.*, 1959

[2] L. Greengard, V. Rokhlin A fast algorithm for particle simulation, in *J. Comput. Phys.*, 1987

[3] A. Elad, R. Kimmel, On bending invariant signatures for surfaces, in *PAMI*, 2003

[4] A. Agarwal, J.M. Phillips, S. Venkatasubramanian, Universal Multi-Dimensional Scaling, in *Int. Conf. Knowledge Discovery and Data Mining*, 2010

[5] D. Boscaini, R. Girdziusas, M.M. Bronstein, Coulomb shapes: using electrostatic forces for deformation invariant shape representation, in *3DOR*, 2014