UNDERSTANDING BLIND DECONVOLUTION VIA VARIATIONAL BAYES

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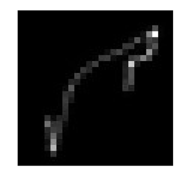
ABSTRACT

Blind deconvolution is a problem of removing blur from a single degraded image without knowledge of the blur process. Two corner stones of effective deconvolution are proper estimator and correct image prior. Many successful methods rely on ad-hoc parameter adjustment or other tricks. We attempt to provide rigorous explanation of these tricks in the context of variational Bayesian estimation with the automatic relevance determination distribution as image prior, where such steps arise naturally.

IMAGE DEGRADATION MODEL

$$g = h * u + n$$

 $=$  $*$  $+$ noise

g..observed image, u..sharp image, h..blur PSF, n..Gaussian noise

ARD IMAGE PRIOR

Automatic relevance determination (ARD) – distribution of image derivatives with variable precision per pixel:

$$p(u|\lambda) = \prod_i \mathrm{N}(D_i u | 0, \lambda_i^{-1})$$

$$\propto \prod_i \lambda_i^{1/2} \left(-\frac{\lambda_i}{2} (D_i u)^2 \right), \quad \lambda_i \ \ldots \ \text{precision at } i\text{th pixel}$$

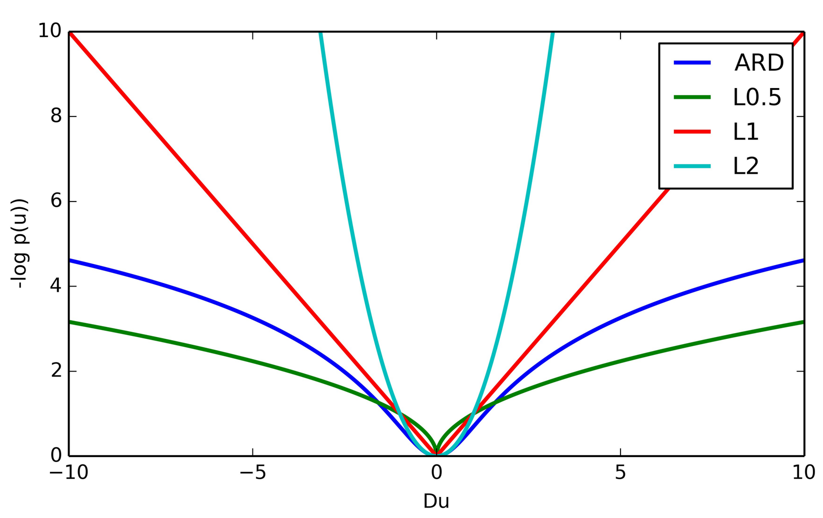


Fig: ARD potential, comparison with L^p.

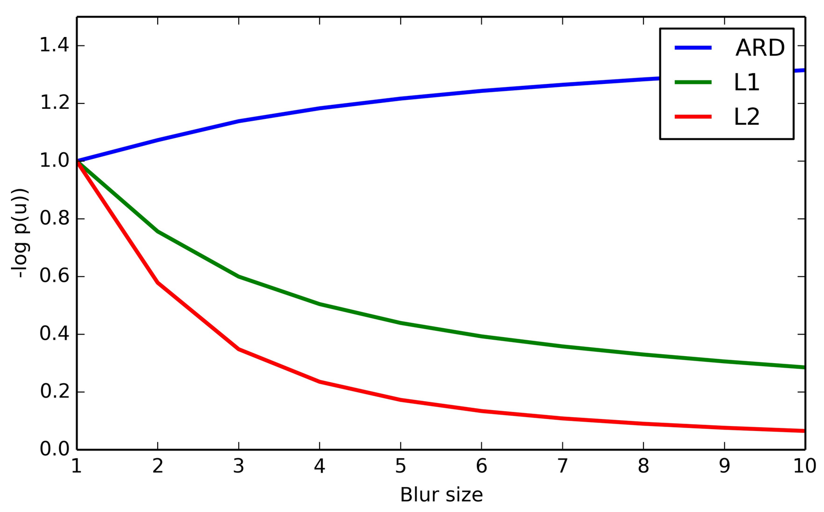


Fig: ARD energy increases with blur size, unlike derivative L^p norm energy.

MAP ESTIMATION

$(\hat{u}, \hat{h}) = \operatorname{argmax}_{u,h} p(u, h|g), \quad p(u, h|g) = p(g|u, h)p(u)p(h),$ where

$p(g\|u, h)$	noise distribution with precision γ
$p(u)$	image prior distribution
$p(h)$	blur prior distribution

BLUR PRIOR

Fixed-precision Gaussian with forced positivity on mean values:

$$p(h|v, \beta) = \prod_i \mathrm{N}(h_i | v_i, \beta^{-1})$$

$$\propto \prod_i \beta^{1/2} \exp\left(-\frac{\beta}{2} (h_i - v_i)^2 \right)$$

$$p(v_i) \propto \exp\left(\psi(v_i)\right), \quad \psi(v_i) = \begin{cases} \infty & v_i < 0 \\ 0 & v_i \geq 0. \end{cases}$$

VARIATIONAL BAYES APPROXIMATION

Approximates the posterior by simpler factorized distribution:

$$p(g, \mathcal{Z}) \approx q(u)q(h)q(\gamma)q(\lambda)q(v)q(\beta)$$

General update equation:

$$\log q(\mathcal{Z}_l) \propto \mathbb{E}_{k \neq l}[\log p(g, \mathcal{Z})]$$

Mean values and covariances of $u, h, \gamma, \lambda, v, \beta$ are updated in alternating manner.

AUTOMATIC NOISE LEVEL ESTIMATION

VB iteratively converges to the correct image noise γ^{-1}, which needs to be manually adjusted in each iteration of many other BD methods.

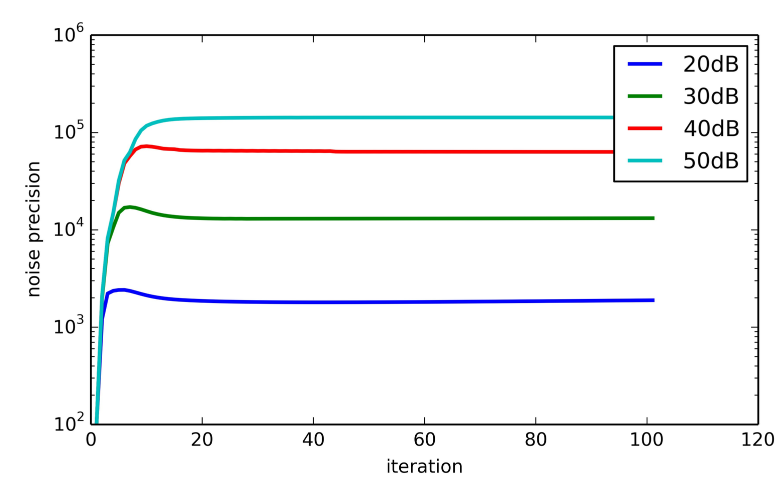
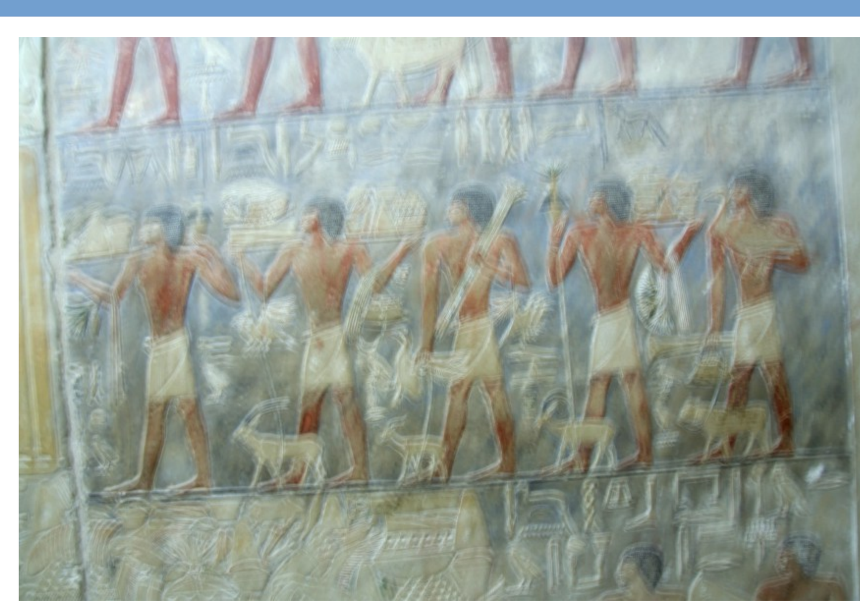


Fig: Noise (data term) precision automatically increases during iterations and saturates on the correct value.

RESULTS

Fig: Real photo blurred by incorrect focus (left) and blind deconvolution result (right) with estimated blur PSF superimposed.



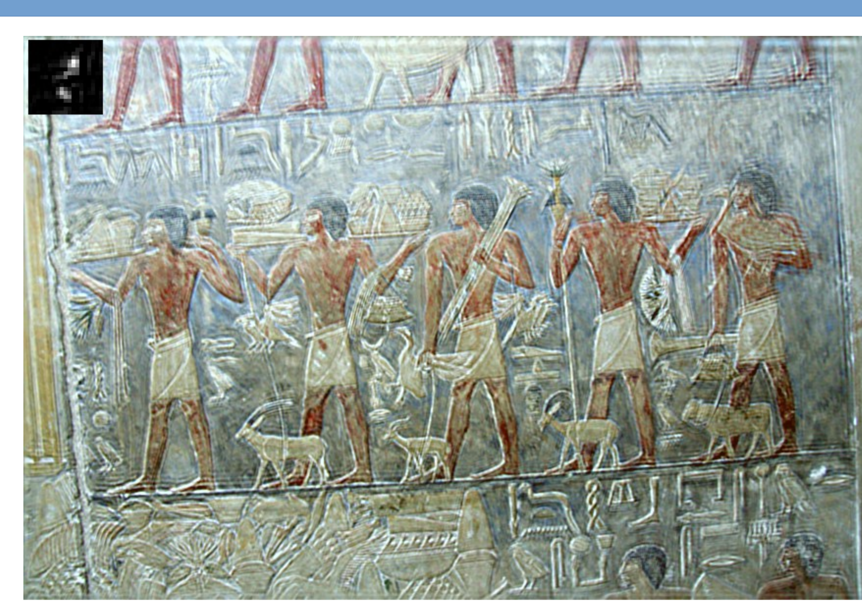


Fig: Real photo blurred by motion blur (left) and blind deconvolution result (right) with estimated blur PSF superimposed.