ACTIVE STRUCTURE ESTIMATION FROM KNOWN MOTION

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Abstract

Structure estimation from motion is a classical topic in computer/robot vision. We propose an active strategy that enforces an estimation dynamics equivalent to that of a linear 2nd-order system with desired poles by suitably acting on the estimation gains and on the inputs applied to the system. This can also be combined with execution of a visual servoing task exploiting a novel projection operator to increase robot redundancy. The theory is experimentally validated in various case studies.

Observer design

Let $(\boldsymbol{s}, \boldsymbol{\chi}) \in \mathbb{R}^{m+p}$ with

$$\begin{cases} \dot{\boldsymbol{s}} = \boldsymbol{f}_m(\boldsymbol{s}, \boldsymbol{u}, t) + \boldsymbol{\Omega}^T(t)\boldsymbol{\chi} \\ \dot{\boldsymbol{\chi}} = \boldsymbol{f}_u(\boldsymbol{s}, \boldsymbol{\chi}, \boldsymbol{u}, t) \end{cases}$$

with input $\boldsymbol{u}$, $\boldsymbol{s} \in \mathbb{R}^m$ *measurable* and $\boldsymbol{\chi} \in \mathbb{R}^p$ *unmeasurable*. The *observer*

$$\begin{cases} \dot{\hat{\boldsymbol{s}}} = \boldsymbol{f}_m(\boldsymbol{s}, \boldsymbol{u}, t) + \boldsymbol{\Omega}^T(t)\hat{\boldsymbol{\chi}} + \boldsymbol{H}\boldsymbol{\xi} \\ \dot{\hat{\boldsymbol{\chi}}} = \boldsymbol{f}_u(\boldsymbol{s}, \hat{\boldsymbol{\chi}}, \boldsymbol{u}, t) + \alpha\boldsymbol{\Omega}(t)\boldsymbol{\xi} \end{cases}$$

with $\boldsymbol{\xi} = \boldsymbol{s} - \hat{\boldsymbol{s}}$, $\boldsymbol{H} > 0$, $\alpha > 0$ (gains) is (locally) exponentially stable iff [1] the Persistence of Excitation (PE) holds

$$\int_t^{t+T} \boldsymbol{\Omega}(\tau)\boldsymbol{\Omega}^T(\tau)\, d\tau \geq \gamma \boldsymbol{I}_p > 0 \ \forall t \geq t_0$$

or (when $m \geq p$) if $\boldsymbol{\Omega}(t)\boldsymbol{\Omega}^T(t) \geq \frac{\gamma}{T}\boldsymbol{I}_p$

The active strategy

- Eigenvalues σ_1^2 of $\boldsymbol{\Omega}\boldsymbol{\Omega}^T$ determine convergence rate
- in SfM $\boldsymbol{\Omega} = \boldsymbol{\Omega}(\boldsymbol{s}, \boldsymbol{u} = (\boldsymbol{v}, \boldsymbol{\omega}))$ and

$$\dot{(\sigma_i^2)} = \boldsymbol{J}_{u,i}\dot{\boldsymbol{u}} + \boldsymbol{J}_{\boldsymbol{s},i}\dot{\boldsymbol{s}}$$

we can *optimize* the behavior by actively choosing $\boldsymbol{u}$ with, e.g.

$$\dot{\boldsymbol{u}} = \frac{k_1 \boldsymbol{u}}{\|\boldsymbol{u}\|^2}\left(\|\boldsymbol{u}_0\|^2 - \|\boldsymbol{u}\|^2\right) + k_2\left(\boldsymbol{I} - \frac{\boldsymbol{u}\boldsymbol{u}^T}{\|\boldsymbol{u}\|^2}\right)\left(\boldsymbol{J}_{u,1}^T - \boldsymbol{J}_{u,1}^{\dagger}\boldsymbol{J}_{s,1}\hat{\boldsymbol{s}}\right)$$

Result:

- fastest convergence speed for a given limited $\|\boldsymbol{u}\|$
- predictability of $\boldsymbol{z} = \boldsymbol{\chi} - \hat{\boldsymbol{\chi}}$ transient (approximates a 2-nd order system)
- online strategy (no pre-planning)

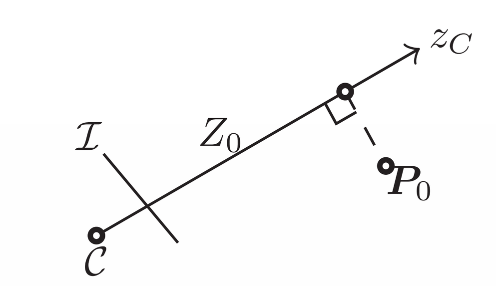
Case studies

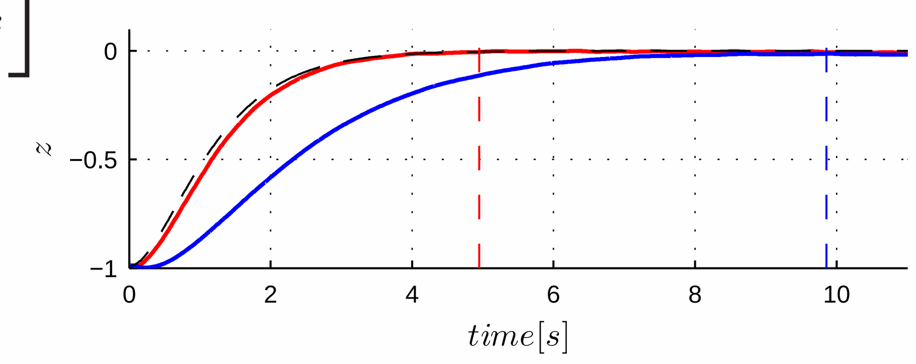
- Point feature [2]: let $\boldsymbol{s} = \left(\frac{X_0}{Z_0}, \frac{Y_0}{Z_0}\right)$ and $\chi = \frac{1}{Z_0}$ with $m = 2 > p = 1$

$$\begin{bmatrix} \dot{s}_x \\ \dot{s}_y \end{bmatrix} = \begin{bmatrix} -\frac{1}{Z_0} & 0 & \frac{s_x}{Z_0} & s_x s_y & -(1+s_x^2) & s_y \\ 0 & -\frac{1}{Z_0} & \frac{s_y}{Z_0} & 1+s_y^2 & -s_x s_y & -s_x \end{bmatrix} \boldsymbol{u}$$

$$\begin{cases} \dot{\boldsymbol{s}} = \begin{bmatrix} s_x s_y & -(1+s_x^2) & s_y \\ 1+s_y^2 & -s_x s_y & -s_x \end{bmatrix} \boldsymbol{\omega} + \begin{bmatrix} s_x v_z - v_x \\ s_y v_z - v_y \end{bmatrix} \chi \\ \sigma_1^2 = \boldsymbol{\Omega}\boldsymbol{\Omega}^T = (s_x v_z - v_x)^2 + (s_y v_z - v_y)^2 \end{cases}$$

note: in all cases only linear velocity $\boldsymbol{v}$ enters in σ_1^2



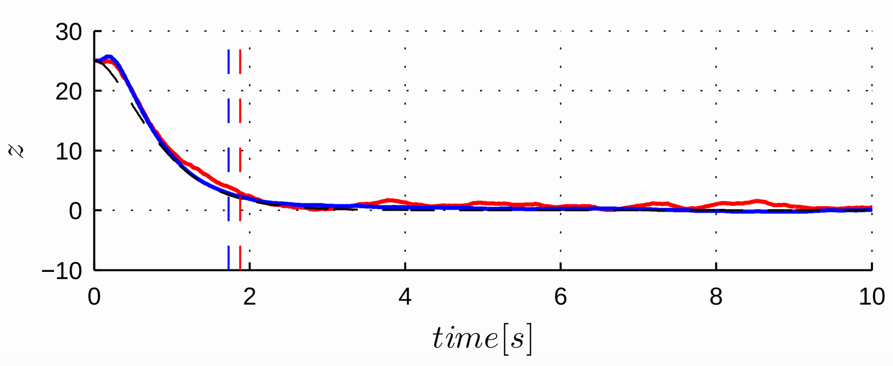


Estimation error for "active" (red) and constant (blue) $\boldsymbol{v}$ (same norm)

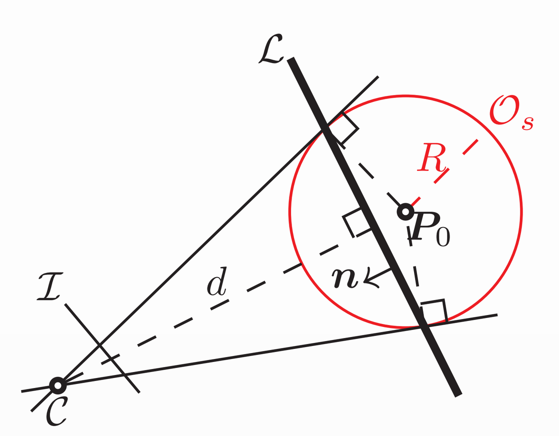
- Spherical target [2]: $\boldsymbol{s} = \frac{\boldsymbol{P}_0}{R}$ and $\chi = \frac{1}{R}$ with $m = 3 > p = 1$

$$\begin{cases} \dot{\boldsymbol{s}} = [\boldsymbol{s}]_\times \boldsymbol{\omega} - \boldsymbol{v}\chi \\ \sigma_1^2 = \boldsymbol{\Omega}\boldsymbol{\Omega}^T = \|\boldsymbol{v}\|^2 \end{cases}$$

note: direction of motion doesn't matter in this case

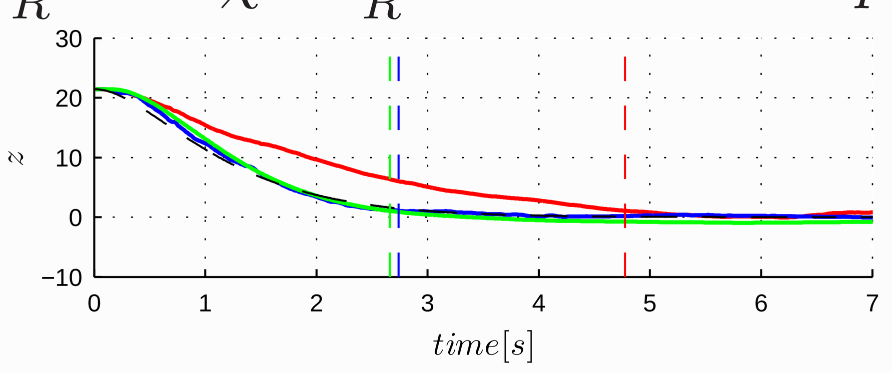


Estimation error for different $\boldsymbol{v}$ (same norm)

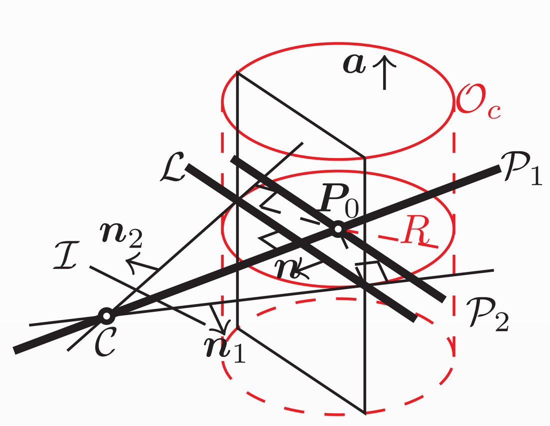


- Cylindrical target [2]: $\boldsymbol{s} = \frac{\boldsymbol{P}_0}{R}$ and $\chi = \frac{1}{R}$ with $m = 3 > p = 1$

$$\begin{cases} \dot{\boldsymbol{s}} = [\boldsymbol{s}]_\times \boldsymbol{\omega} + (\mathbf{a}\mathbf{a}^T - \boldsymbol{I})\boldsymbol{v}\chi \\ \sigma_1^2 = \boldsymbol{\Omega}\boldsymbol{\Omega}^T = \|\boldsymbol{v}\|^2 - (\mathbf{a}^T\boldsymbol{v})^2 \end{cases}$$



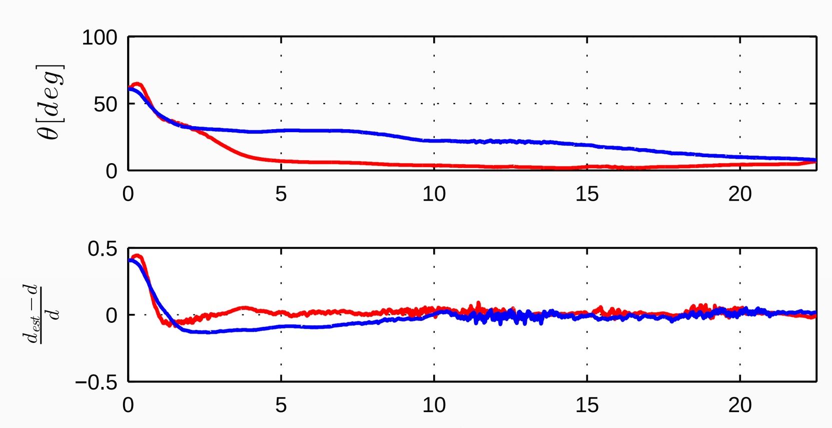
Estimation error for "active" (blue and green) and constant (blue) $\boldsymbol{v}$ (same norm)



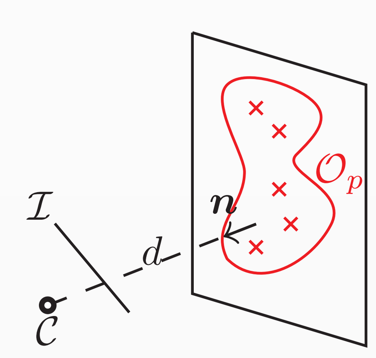
- Planar scene [3]: $\boldsymbol{s} = (m_{ij})$ with $m_{ij} = \sum_{k=1}^{N} x_k^i y_k^j$ (discrete points) or $m_{ij} = \iint_{\mathcal{O}_p} x^i y^j \, dx dy$ (dense patch) and $\boldsymbol{\chi} = -\boldsymbol{n}/d$ with $m \geq p = 3$

$$\begin{cases} \dot{m}_{ij} = f_{m_{ij}}(m_{kl}, \boldsymbol{\omega}) \\ \qquad + \boldsymbol{f}_{\Omega_{ij}}(m_{kl}, \boldsymbol{v})\boldsymbol{\chi} \\ \sigma_{1,2,3}^2 = \sigma_{1,2,3}^2(m_{kl}, \boldsymbol{v}) \end{cases}$$

expression depends on chosen moments but can always be computed in closed form



Estimation error for "active" (red) and constant (blue) $\boldsymbol{v}$ (same norm)

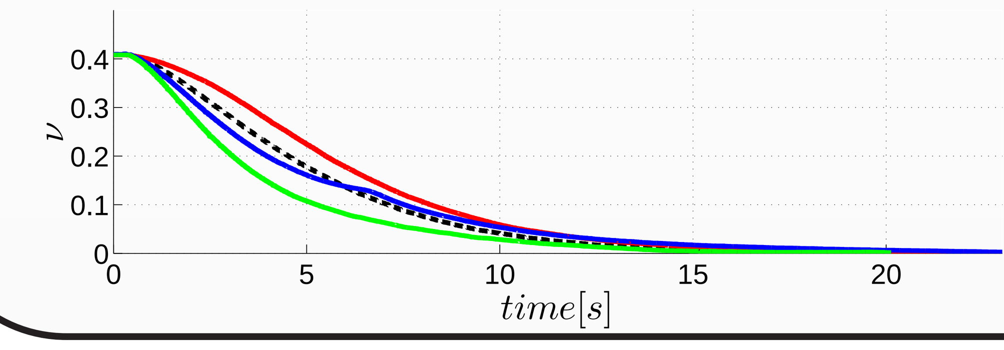


Multitask coupling [4]

In visual servoing we often have:

$$\frac{d}{dt}(\text{task}) = \dot{\boldsymbol{r}} = \boldsymbol{J}(\boldsymbol{s}, \boldsymbol{q}, \boldsymbol{\chi})\boldsymbol{u}$$

- estimating $\boldsymbol{\chi}$ while executing the task can improve performance
- redundancy can be exploited for active estimation
- redundancy can be maximized controlling task error norm ν (one quantity) instead of task error



References

[1] A. De Luca, G. Oriolo, and P. Robuffo Giordano, "Feature depth observation for image-based visual servoing: Theory and experiments," *Int. Journal of Robotics Research*, vol. 27, no. 10, pp. 1093–1116, 2008.

[2] R. Spica, P. Robuffo Giordano, and F. Chaumette, "Active Structure from Motion: Application to Point, Sphere and Cylinder," *IEEE Trans. on Robotics*, conditionally accepter.

[3] ——, "Experiments of Plane Estimation by Active Vision from Point Features and Image Moments," in *submitted to 2014 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, Chicago, IL, Sep 2014.

[4] ——, "Coupling Visual Servoing with Active Structure from Motion," in *2014 IEEE Int. Conf. on Robotics and Automation*, Hong Kong, China, May 2014.